[Tutorial] - Quantum Memory:
Superconducting qubits and Quantum Computer Hardware Design

Part #1
Parr #2

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Quantum Technology, LLC
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Professional Development Series

the Future of Memory and Storage

What is Quantum Computer?

Quantum Computers represent a fundamentally new paradigm for processing information

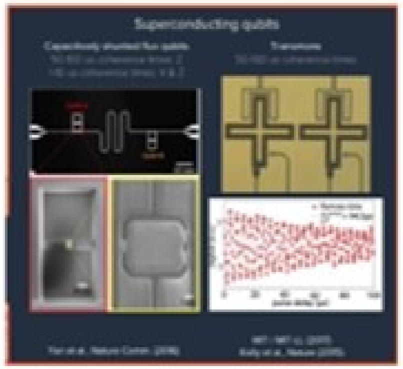
- Exceed Performance of Conventional Computer
- Quantum Advantages
  - Solved a problem on a quantum computer, the problem are hard for classical computer.
- Quantum Computer vs. Conventional Computers

Ref. : MIT Quantum Computing Fundaments, MIT/xPRO

Quantum Computer:

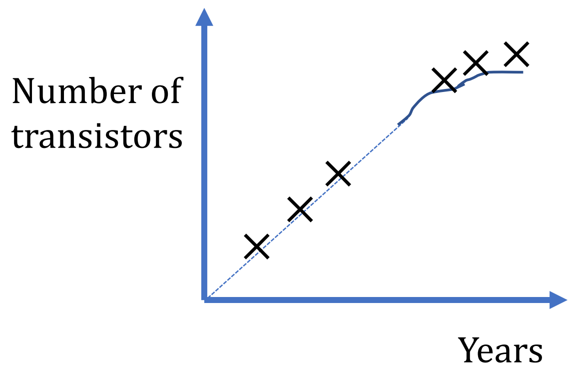
Source: MIT/Lincoln Laboratory Quantum Computer (Super-Conducting Qubits)

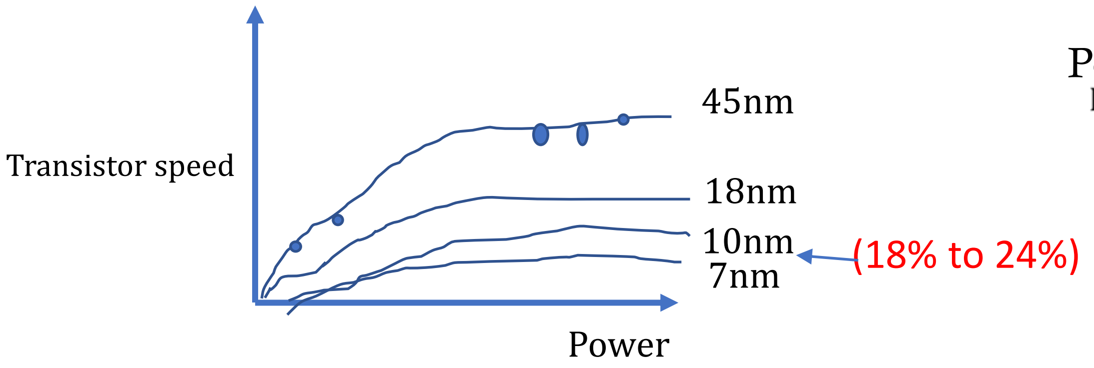
1945– Computer, ENIAC

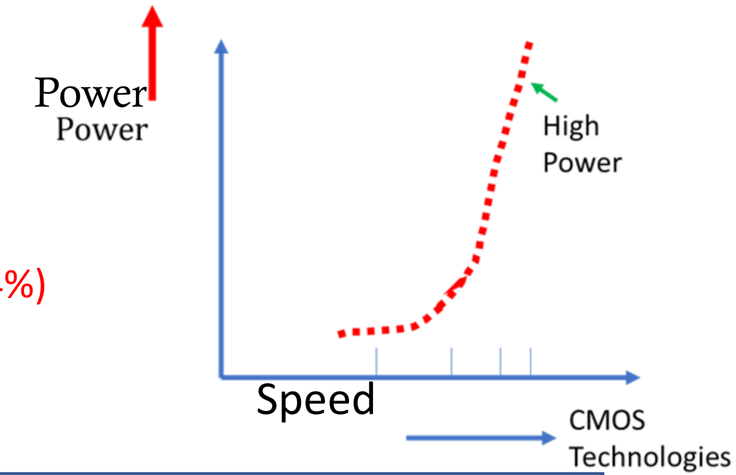


Quantum Computing?

- Richard Feynman and Marnin (USSR) proposed Quantum Computer to simulate Quantum Mechanics (1981)
- Transistor scaling is slowing down. Lacking Innovation?
- Building a new Fab requires $20 billion (GDP size of budget).

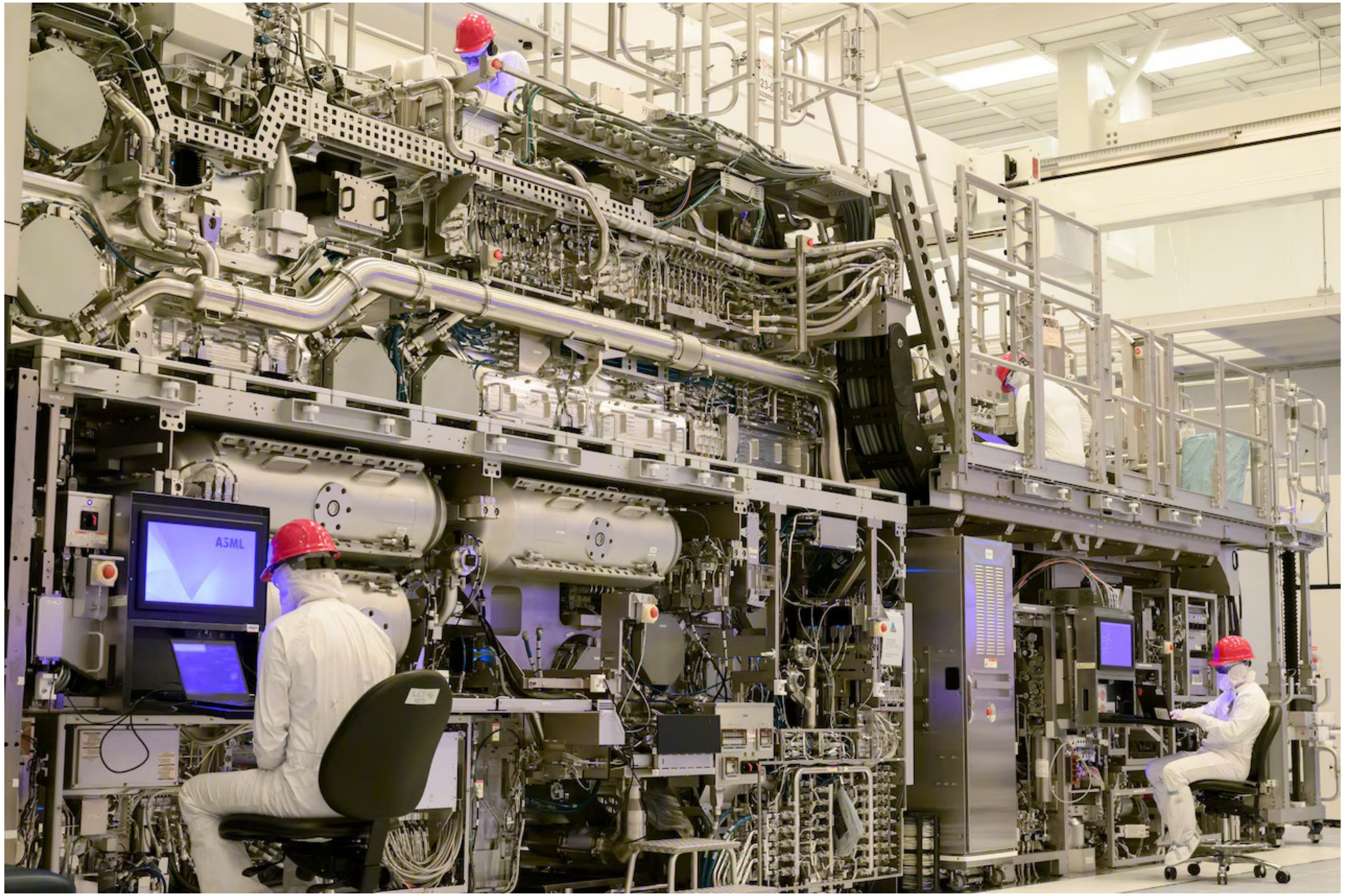






- Application requirements
  - Chemistry Simulation- Medicine
  - AI- Machine learning
  - Optimization problems

Why Quantum Computer?



A High NA EUV manufacturing tool inside Intel's D1X research factory in Hillsboro.

Challenges– CMOS Technology

- Logic and Memory silicon's performance is solely based on CMOS scaling down.– followed Moore's law
- Transistor slows down when CMOS transistor reached 10nm and below.
- High manufacturing cost, limited applications can use the technology below 10nm.
  - impacting on new(invented) ideas due to high cost and limited wafer suppliers
- CMOS silicon chips' power increased exponentially when used below 10nm technologies
- New computer architectures without relying on transistor scaling down.

Economy and Technology

FMS the Future of Memory and Storage

Quantum Computing Progress and Opportunities:

- Classical Computer
  - 40 years--invention of the vacuum tube in 1906 to first vacuum tube base computer.
  - 25 years– invention of the transistor in 1947 to first commercial integrated circuit (IC) chips, Intel – 4004(1971), 8008(two years later).
- Quantum Computer
  - 1980– Richard Feynman suggested Quantum computer to simulate Quantum system
  - Mid-1990– Peter Shor, Shor's Algorithm is the first algorithm solved a practical problem, **Factorization of large number** (Ex. 15 = 3 x 5)
    - Factorization is hard problem for classical computer
  - Shor, Robert Calderbank, and Andrew Steen– First Quantum error correction codes.

Quantum Computing Progress and Opportunities(2):

- 20 to 30 years
  - Quantum computer with 1,000,000 Qubit fault-tolerant machine
- Near- Term commercial application of quantum information technology-- (NISQ)
  - Noisy, intermediate-scale Quantum simulation
  - Noisy, intermediate-scale Optimization
- Quantum Utility– 133 qubits to 1000 qubits (source: IBM)
  - The era of Quantum Utility—Hardware and Software
  - The quantum computer run circuits beyond the reach of classical simulations
- Opportunities
  - Various components generates new business opportunities
  - Optical, Electronics, Software, and Refrigeration

The history of Quantum Computing

- 1900 – 1930 Quantum Mechanics
- 1936 Einstein, Podoski and Rosen (EPR): “ Quantum Mechanics is in complete”
- 1936 Schrodinger: Entangle Particles
- 1964 Bell Proved that no classical explanation for the behavior of EPR pair. So Quantum Mechanics is incomplete not in a classical way, QM is weird.
- 1980 Aspect showed Bell’s predictions were correct. Many peoples showed that no classical explanation for Q.M. Quantum Mechanics works.
- 1982 Herbert, FLASH, “Faster than light communication using weird properties of QM and EPR Paris.”
- 1982 Two groups of peoples found out why Herber’s paper was wrong. No-Cloning Theorems, i.e. A single unknown Quantum state cannot duplicated.
- 1982 Richard Feynman and Manin (USSR)proposed Quantum Computer to simulate Quantum Mechanics.

Source: Peter Shor’s class notes

The history of Quantum Computing(2)

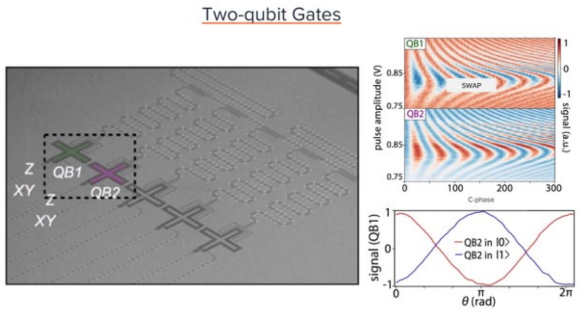
- 1985 David Deutsch described Q. Machine
- 1992 Deutsch and Josza Algorithm
- 1993 Bernstein-Vazirani Problem
- 1994 D. Simon Algorithm (Simon problem) demonstrated the Algorithm is exponential faster than classical computer. Simon algorithm has no practical application. Simon Algorithm motivates Shor's discovery of the famous Quantum Algorithm for Periods finding for Factoring,
- 1994 Peter Shor (Bell Lab./MIT), Shor's Algorithm for Factoring , Quantum Computing field took-off. Shor's Algorithm is super efficiency Quantum Algorithm for finding periods for factoring large numbers.
- 1995 Lov Grover, Search Algorithm

Quantum Computer Applications:

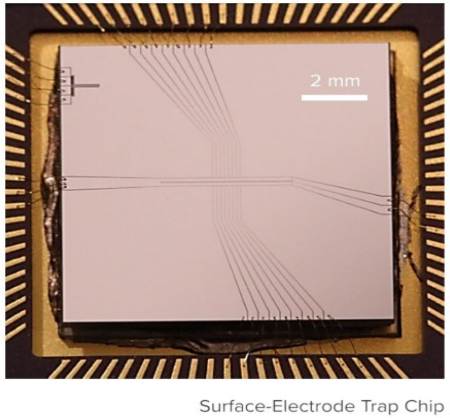
- Cybersecurity– Quantum communication
- Materials Science – New Battery technology
- Chemistry– Nitrogen Fixation(Fertilizer)
- Pharmaceuticals– Genome Sequencing
- Machine learning– Artificial Intelligence(AI)
- Optimization
- more

Qubits Technology Summary:

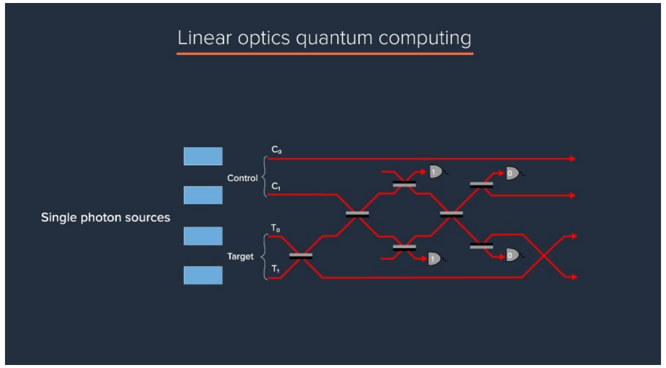
- Superconducting Qubits
- Trapped Ion
- Topologic Qubits
- NV centers
- Photonic
    - Non-solid state platform)
- Silicon



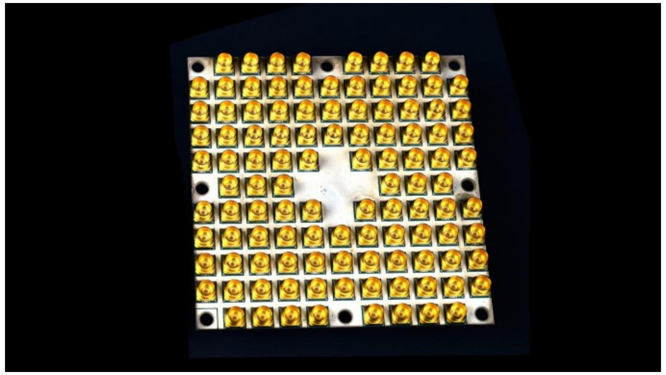
Superconductor



Trapped Ion



Photonic

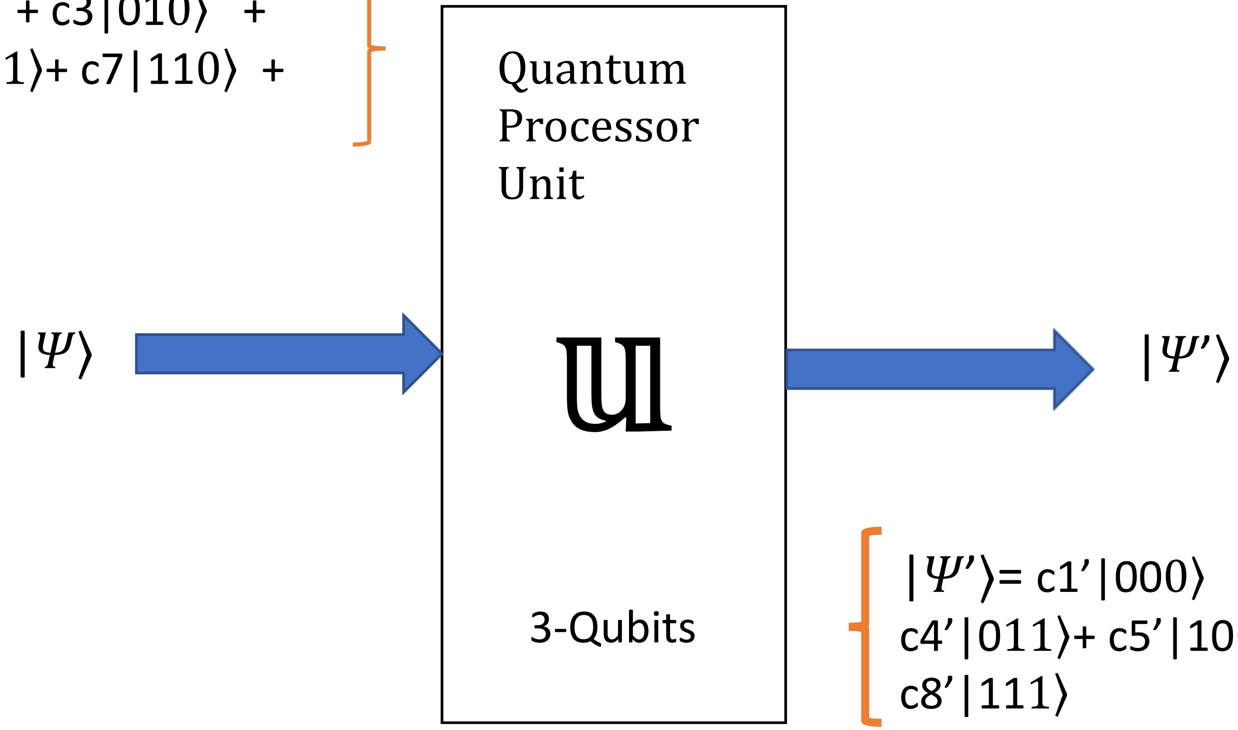


Silicon

Sources: Intel and MIT-Lincoln Lab.

Qubits Technology	Number of qubits	T1/T2 time(ns) Fidelity Readout time	Scalability	Advantages	Disadvantages
Superconductors	433	50us/100us 99.9% 5MHz	Possible, qubit size, and scale of integration CMOS compatibly	Well researched technology CMOS compatible process Conventional control equipment,	Low coherence time, fast gate Sensitivity to noise Low temperature(15 to 20mK)
Trapped Ions	53	> $1e^{14}$ (Years)/50s 99.0% $1.00e^{-4}$MHz	Difficult, High level of integration is difficult. CMOS compatibly	Good stability Long coherence time, slow gate operation 4K to 10K temperature Laser as control equipment	Too slow, slow quantum calculation
Photon	20		Yes, Silicon technology	High operating temperature CMOS technology, photons are using in telecom	High error rate, No possibility to store photons
Silicon(SOI, SIGe)		1000ms/0.4ms 99.6% 1MHz	Yes, Silicon technology	CMOS technology, Fast quantum gates,	
NV Centers		100ms/200ms 94% $2.0e^{-02}$ MHz		High temperature (4K) Long coherence time Used as memory	Complex scalability
Quasi particles (Anyon, fermions de Majorana)			May be, if it is semiconductor technology		

$$\left\{ |\Psi\rangle = c1|000\rangle + c2|001\rangle + c3|010\rangle + c4|011\rangle + c5|100\rangle + c6|101\rangle + c7|110\rangle + c8|111\rangle \right\}$$

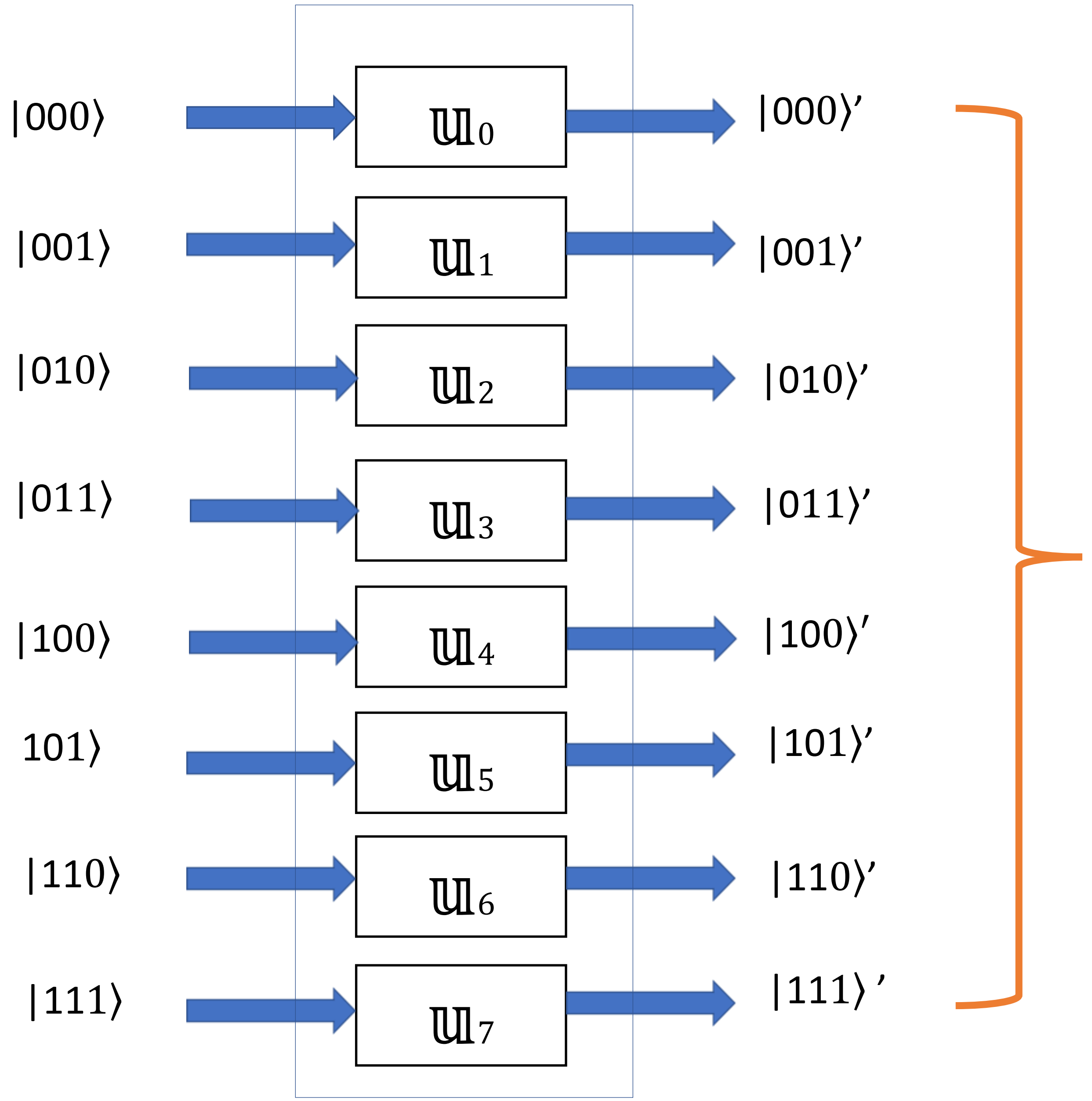


$$\left\{ |\Psi'\rangle = c1'|000\rangle + c2'|001\rangle + c3'|010\rangle + c4'|011\rangle + c5'|100\rangle + c6'|101\rangle + c7'|110\rangle + c8'|111\rangle \right\}$$

Parallelism

Interference

Digital Computer Units



Multiple Resources

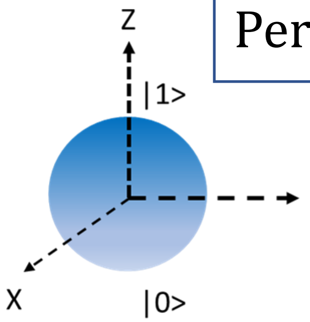
N = 3 bits

Number of Qubits

Quantum Computer Performance

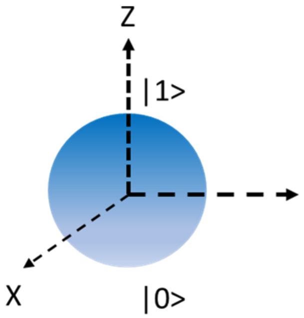
Classical Computer Performance

30 Qubits



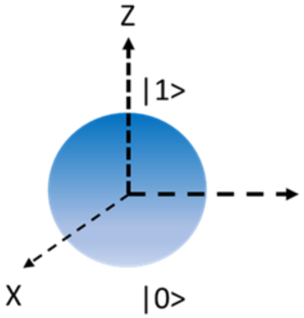
(PC)

40 Qubits



(Supercomputer)

80 Qubits



(All the computers on earth?)

Qubits

Quantum Advantages

Classical Computer : Takes average two and half (2.5) steps
Quantum Computer: Takes One step

Search coin:

1.
One tail, three heads

Classical computer:

$2^{N-1} +1 = 3$ (N=2)

2. One tail, seven heads

$2^{N-1} +1 = 5$ (N=3)

Quantum Computer:
One step for all Ns.

(Exponential increase)

Quantum Advantages

Factoring Numbers

- Factoring a large number into primes (Shor's Algorithm)

$$M= p * q \qquad (15= 3 \times 5)$$

Classical computer, $t \sim \exp(\, \mathbf{O}(n^{1/3}\log^{2/3} n))$

28,000,000,000,000,000,000,000 years

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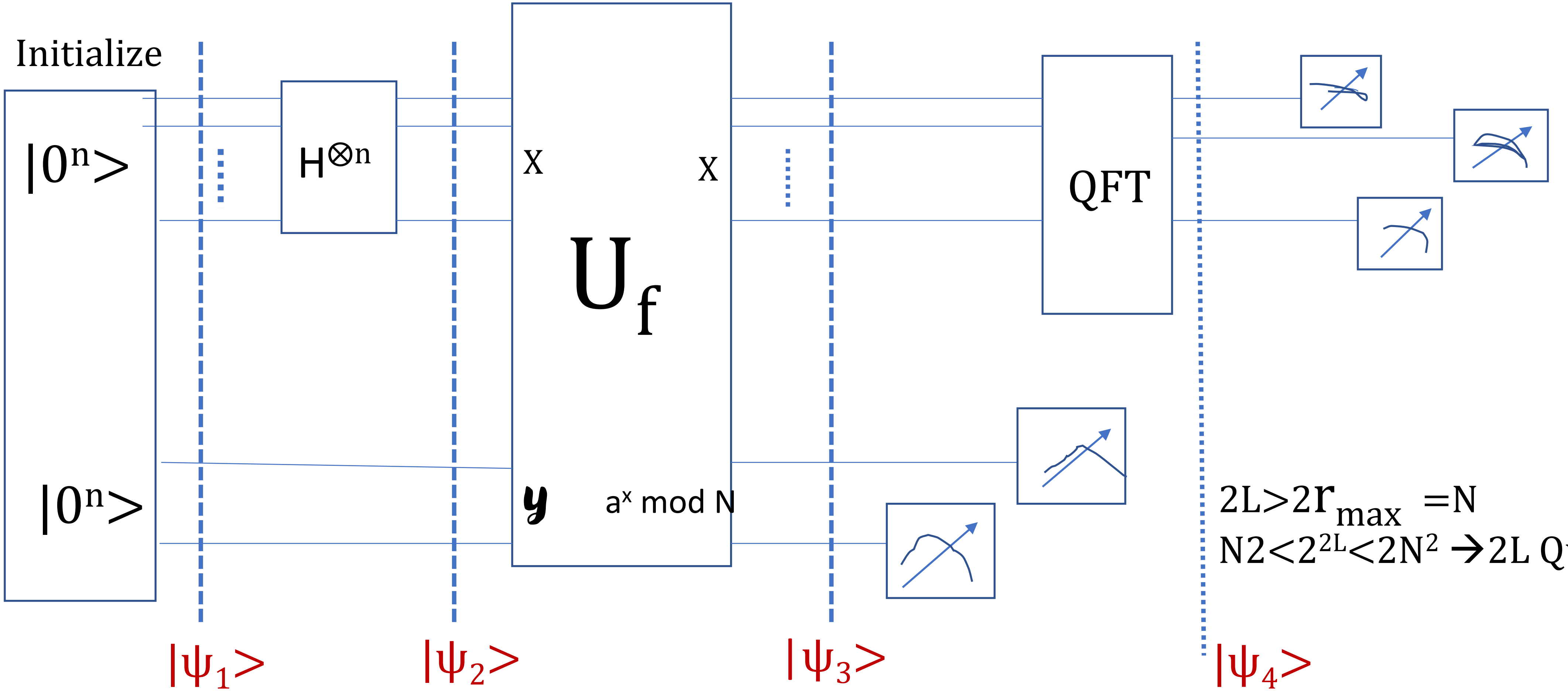
Quantum computer, $t \sim O(n^3)$

100 seconds

Classical computer time increases in exponential speed.
Quantum computer time increases in polynomial speed.

Finding Period -- Quantum Circuit (Example)



$2L > 2r_{max} = N$

$N2 < 2^{2L} < 2N^2 \rightarrow 2L$ Qubits

$|\psi_1 = H^{\otimes n}|0^n> = 1/\sqrt{2}^n \sum_{x=0}^{n-1} |X>$

$|\psi_2> = (1/\sqrt{2^n}) \sum_{x=0}^{2^n-1} |X>|f(X)>$

Universal Quantum Computer

- Universal Quantum Computer allows users to run, implement any Universal Quantum Algorithm.
- Computation Complexity Classes
  - Computational resources, such as memory size scales with the size of complexity of the problem.
- Polynomial scaling– efficient, Ex. $a \times n^{b}$, a, b = constants

  (Class =P) $n^2 = 1,4,9$ (n =1,2,3), $n^2 = 100$
- Exponential scaling– not efficient, Ex. $2^n = 2,4,8$ (n=1,2,3) , $2^{10} = 10,000$

  (Class = NP)

IBM Q System One

Source: Forbes, and IBM

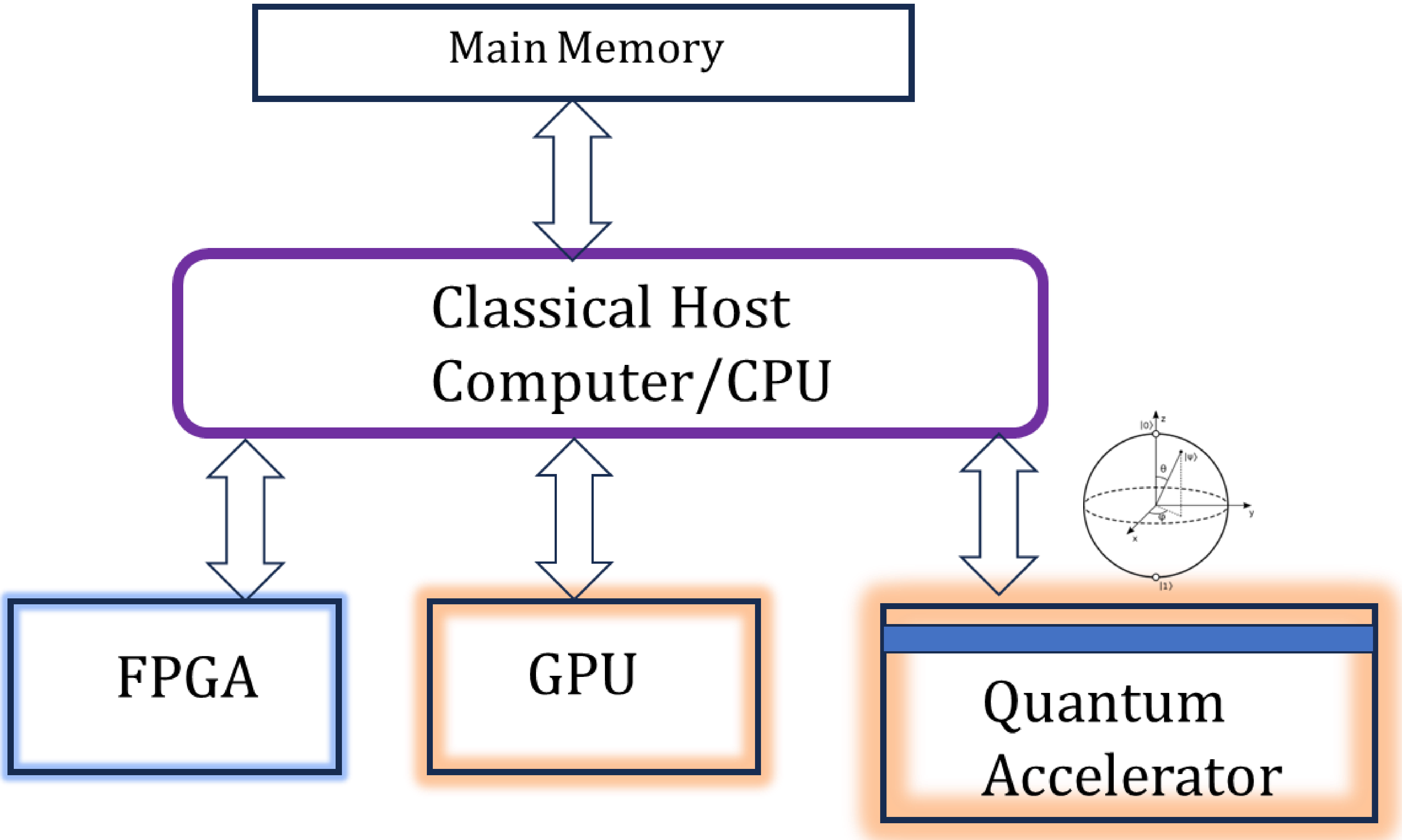
	Classical	Quantum	Quantum advantage	
Fourier Transformation	$O(n2^n)$	$O(n^2)$ $O(nlogn)$[1] gates	Quadratic polynomial in the number of qubits Exponentially speed	N= number of qubits or classical bits
Deutsch-Jozsa Problem	$2^{N-1}+1$ (steps)	1- step	Exponentially speed	
Simon problem	at least $\Omega(2^{n/2})$ queries	O(n) queries to the black box	Exponentially speed	
Shor's Algorithm[2]	$O\left(e^{-1.9(logN)^{1/3}(loglogN)^{2/3}}\right)$ $\sim e^{(logN)^{1/3}}$	$O\left((logN)^2 (loglogN)(logloglogN)\right)$ $\sim(\log N)^3$	Polynomial-time got integer factorization	To factor an integer N
Grover Search Algorithm (1996)	O (N)	$O(\sqrt{N})$		

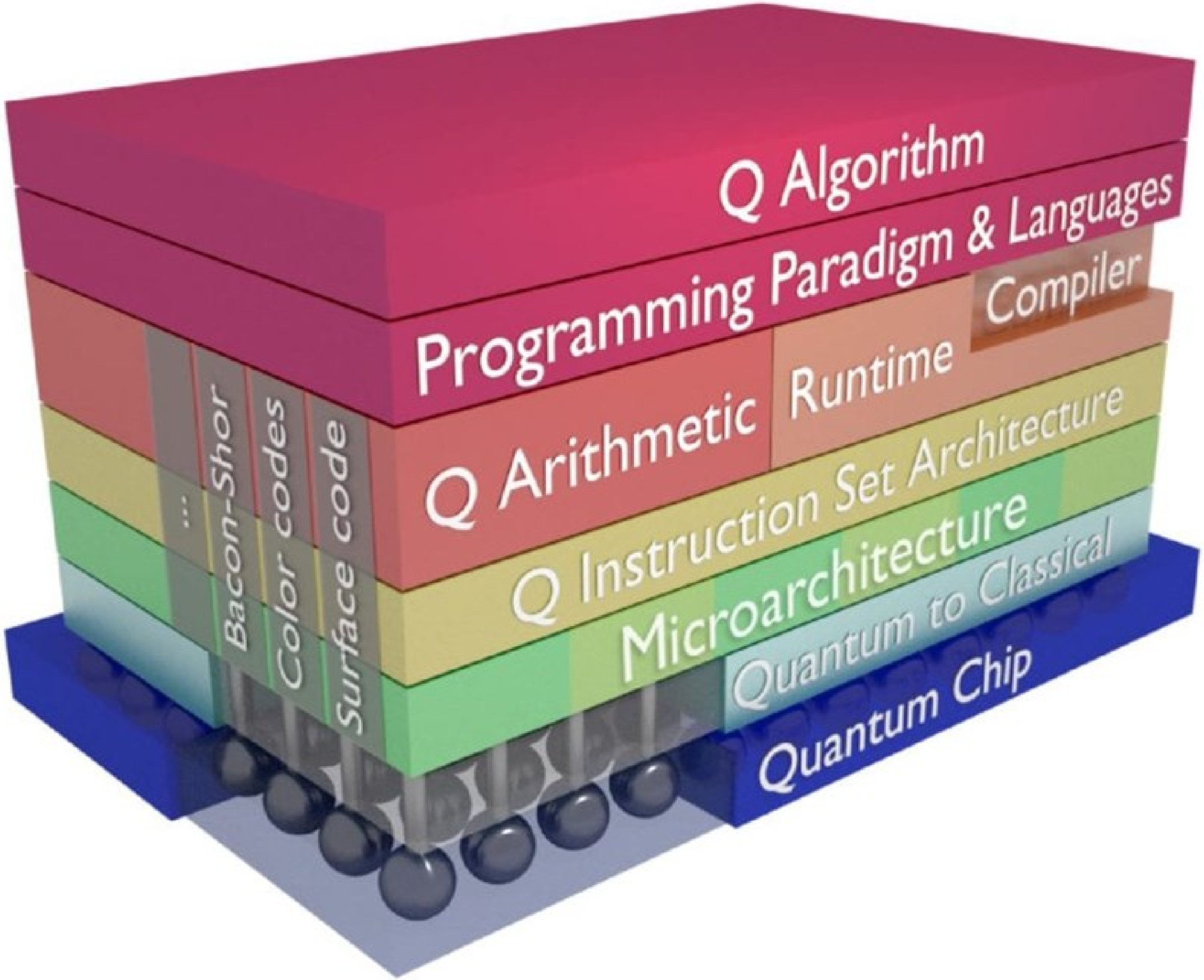
[1] :Late of 2000, the best Quantum Fourier transform algorithm, Wikipedia,
[1a]: Quantum Fourier Transformation discovered by Don Coppersmith, 1994.
[2]: Classical resource is memory and runtime, Quantum resource is physical bits and runtime.

Quantum Simulation (VQE)

- Quantum Computer can simulate many types of simulation problems.
- Hybrid-classical-quantum systems to simulate classical algorithms, one example called a

  variational quantum eigensolver (VQE)
- The Quantum computer is acting as a co-processor. The classical computer and quantum computer are passing information back and forth throughout the simulation.
- To simulate the hydrogen molecule composed two electrons, a quantum processor with 2 qubits.

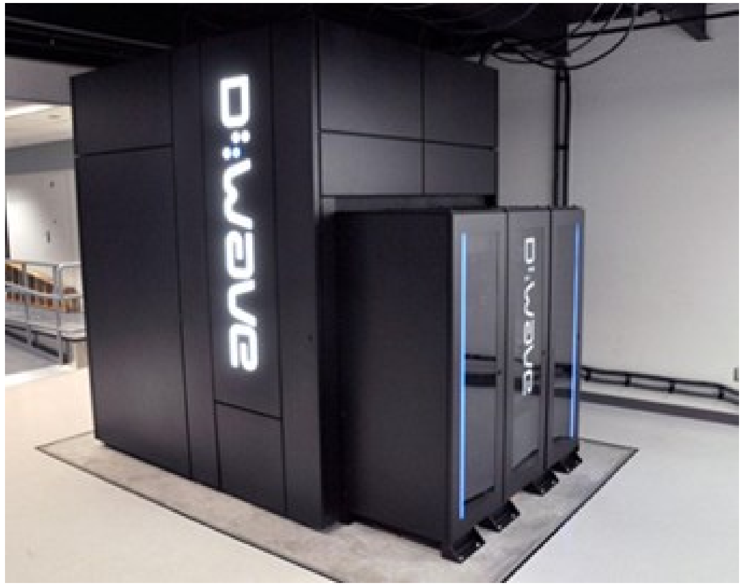

Main Memory
Classical Host Computer/CPU
FPGA
GPU
Quantum Accelerator



Source:
QuTech Academy

Quantum Annealing (Quantum Annealer)

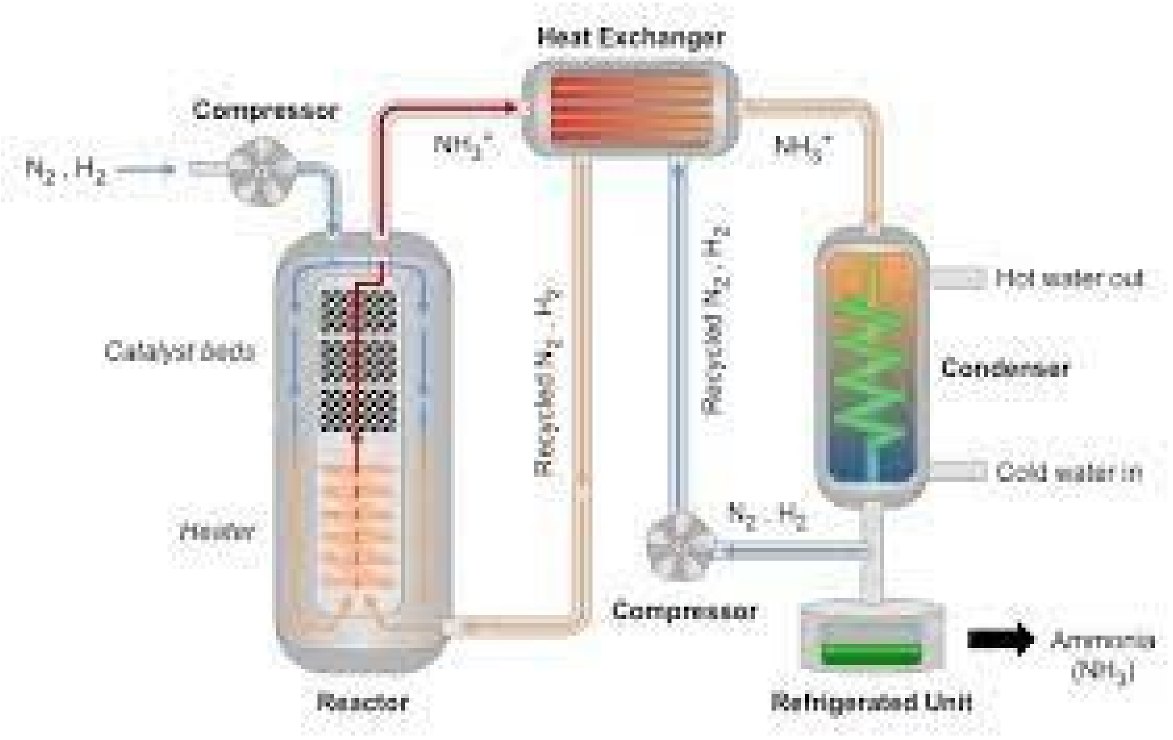
- A quantum annealer is a special application-specific Quantum Computer.
- Addressing Classical optimization problems
- Quantum annealer does not use digital gates, optimization problem is encoded into the qubits.



DWave launches the first Quantum Computer (Quantum Annealing) --2011

Quantum Chemistry Simulation

- Nitrogen fixation
  - Quantum simulation of the Chemical reaction mechanisms
  - Haber Bosch process– needs high temperature and high pressure
  - Chemists know there exist bacteria use an enzyme called molybdenum nitrogenase which at room temperature can catalyze atmospheric nitrogen into ammonia. We don't know how to simulate it.
- Pharmaceutical drugs



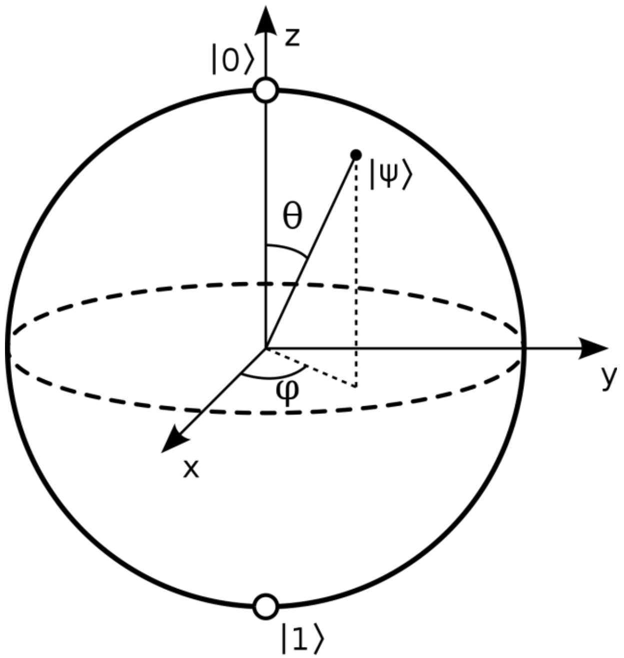
Quantum Communication– Basic concepts

- Quantum Communication is different with Classical communication
    - Encoding, Transmission, Authenticating information
- Quantum Communication basic concepts
    - Superdense coding or Quantum dense coding (QDC)
        - Transporting two bits classical information through a single Qubit
        - Quantum entanglement – two users access pre-shared entangled qubits.
    - No-cloning theorem of Quantum Mechanics
    - Intercept or measure a quantum state are detectable
        - Measurement process leaves a detectable signature

Quantum Hardware– Computer

- Computation must be robust against Noise.
  - Quantum Computers are vulnerable noises, noises generate errors for Quantum Computers
  - Classical computer has many ECC and fault tolerance techniques to correct errors.
    - Check Point, Error Correcting code (ECC), and redundancy
  - ECC is working on Quantum Computer
  - Classical computer does not use ECC due to significant overhead, but memory chips and communications use ECC.
  - Memory Chip's ECC is using a stable Logic circuit of detecting and correcting memory errors only.

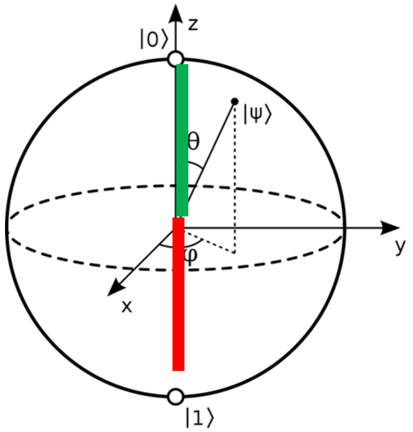
Fundamentals of Quantum Information



Superposition

$|\psi> = \alpha|0> + \beta|1>$

The Born Rule: $|\alpha|^2 + |\beta|^2 = 1$



Inner product

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha \begin{pmatrix}1\\0\end{pmatrix} + \beta \begin{pmatrix}0\\1\end{pmatrix}$$

$$= \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

If $|\alpha|^2 = |\beta|^2 = 1/2$; 50% ; $|0\rangle$, 50%; $|1\rangle$

Superposition

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\quad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

Qubit States

$|\Psi\rangle = |000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle$

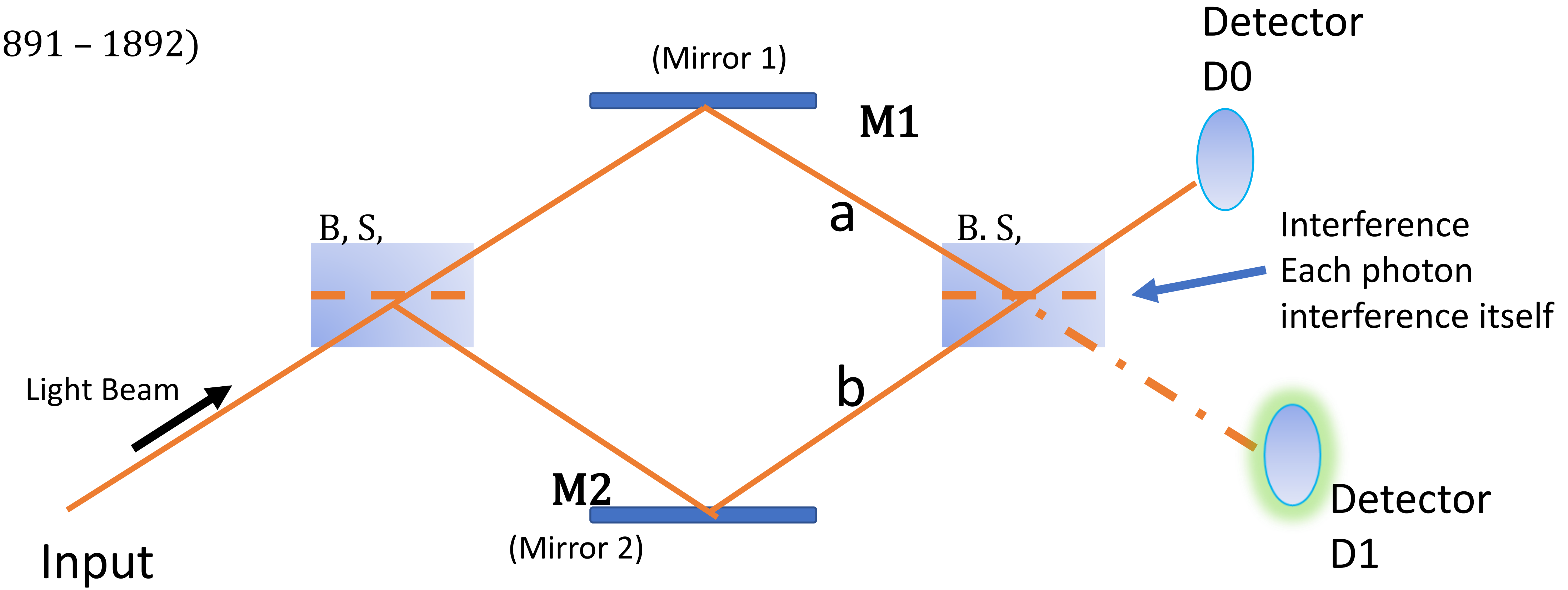
3-bits

Nature of Superposition

- Marc-Zehder Interferometer

(1891 – 1892)

Superposition :Single photon
= Superposition Photon upper beam
Photon lower beam

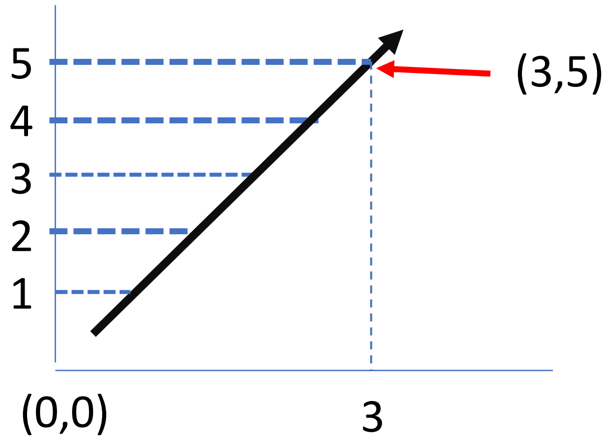


Linear Algebra for Quantum Computing

- Linear Algebra (LA) is the language of Quantum Computing.
- Basic of Linear Algebra

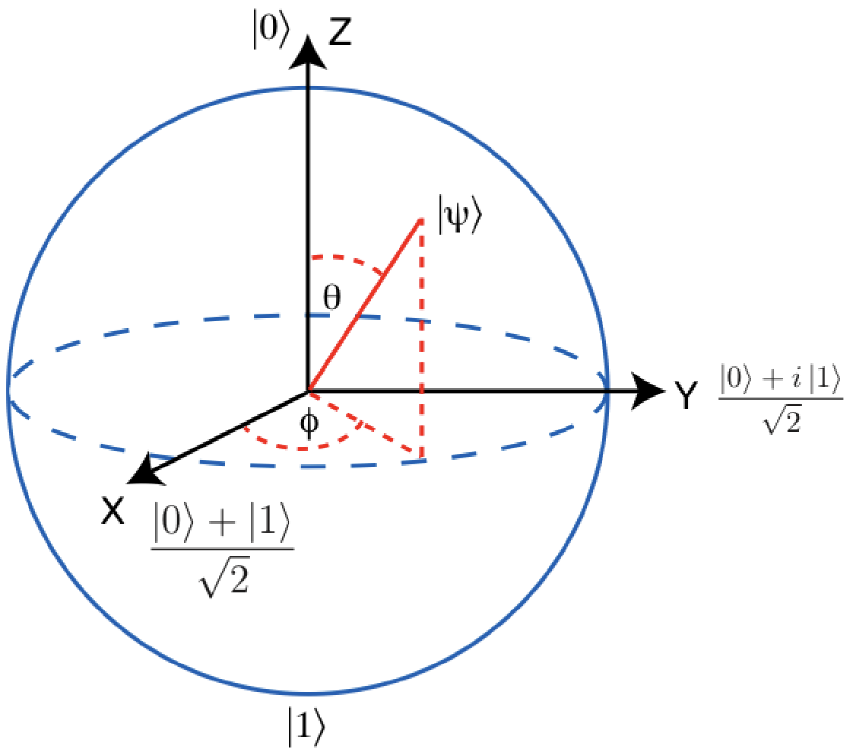
Vector |v>, "is a mathematical quantity with both *direction* and *magnitude*"

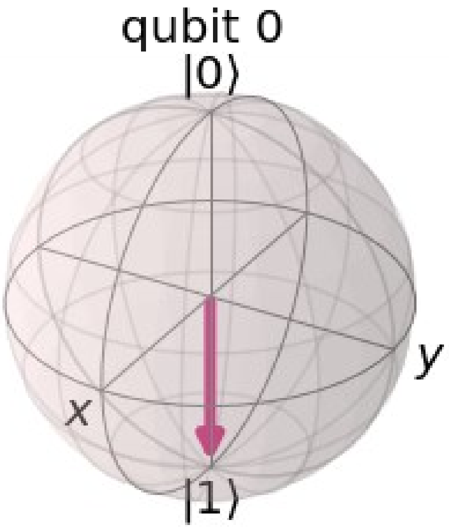
$|v\rangle = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$,



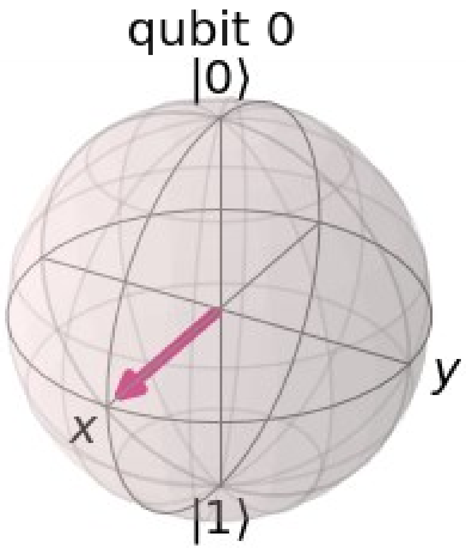
In Quantum Computing, State-vector → corresponds to a specific Quantum State

- # Bloch Sphere





Superposition between |0>, |1>. The arrow is halfway between |0>, at the top, |1> at the bottom.
Arrow can rotate to anywhere surface of the sphere.



https://qiskit.org/textbook/ch-states/introduction.html

Vector Space

$$\begin{bmatrix} x1 \\ y1 \end{bmatrix} + \begin{bmatrix} x2 \\ y2 \end{bmatrix} = \begin{bmatrix} x1 + x2 \\ y1 + y2 \end{bmatrix},$$

$$n|\nu> = \begin{bmatrix} nx \\ ny \end{bmatrix} \in \nu, \forall n \in R, \ |\nu> = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$|a> + |b> = |c>$$

Matrices and Matrix Operation: Matrices are mathematical objects that transform vectors into other vectors,

$$|\nu> \rightarrow |\nu'> = M|\nu>$$

Ket Notation:

Column Vectors → Kets : $|\varphi\rangle = |0\rangle + |1\rangle$

Duel vectors = bras; $\langle\varphi| = [\alpha^* \ \ \beta^*] = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$

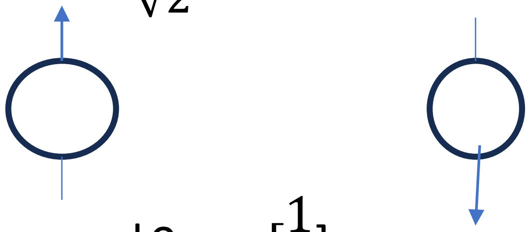
Inner Products = $[\ \alpha^* \ \ \beta^*] \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$

$= \alpha^* \alpha + \beta^* \beta$

$|0\rangle/|1\rangle$ measurement yields $|0\rangle$ with probability $|\langle\varphi|0\rangle|^2 = |\alpha|^2 + |\beta|^2 = 1$

Different basis: $|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$; $\alpha|0\rangle + \beta|1\rangle = (\frac{\alpha+\beta}{\sqrt{2}})\ |+\rangle + (\frac{\alpha-\beta}{\sqrt{2}})\ |-\rangle$

Spin-1/2



$|0\rangle = \begin{bmatrix}1 \\ 0\end{bmatrix}$ $|1\rangle = \begin{bmatrix}0 \\ 1\end{bmatrix}$

Quantum Computation

Basis, $|0\rangle$, $|1\rangle$

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}} \text{ (linear combination)}$$

Superposition of $|0\rangle$ and $|1\rangle$ basis state, equal probability of measuring the state to be in either one of the basis vectors states, $\frac{1}{\sqrt{2}}$

Hilbert space, Inner product, $|a\rangle$, $|b\rangle$ -- Inner product: $\langle a|b\rangle$

$\langle a|$ is the conjugate transpose of $|a\rangle$, $|a\rangle^\dagger$

$$\langle a|b\rangle = [a_1^* \ a_2^* \ \ldots\ldots\ldots \ a_n^*]\begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{bmatrix} = a_1^* b_1 + a_2^* b_2 + \ldots\ldots\ldots + a_n^* b_n$$, Where * = complex conjugate

Ex. $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\langle 0|0\rangle = 1$, $\langle \psi|\psi\rangle = 1$

- Unitary Matrix, $U^{\dagger}U = I$

$$|\psi'> = U|\psi>$$

Ex1. $|\psi>= a|0> + b|1>$, $U= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,

$|\psi'>=U|\psi> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix} = b|0> + a|1>$

Ex2. Let $|\psi>= 1|0> + 0|1> = |0>$, $U=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$|\psi'> = U|\psi> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0> + \frac{1}{\sqrt{2}}|1>$

Ex3. $U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ then $U^{\dagger} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$U^{\dagger}\ U = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = I$

- Tensor Product ➔ |ϕ> ⊗ |φ> -- tensor product, |ϕ>|φ>

Ex. $|\phi\rangle = \begin{bmatrix} 2 \\ 6i \end{bmatrix}$, $|\varphi\rangle = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\langle\phi|\varphi\rangle = [2 \quad -6i] \begin{bmatrix} 3 \\ 4 \end{bmatrix}$=6-24$i$

➔|ϕ>⊗|φ> => |ϕ>|φ>

Ex. $|\phi\rangle|\varphi\rangle = \begin{bmatrix} 2 \\ 6i \end{bmatrix} \otimes \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2\times 3 \\ 2\times 4 \\ 3\times 6i \\ 4\times 6i \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 18i \\ 24i \end{bmatrix}$

A* -- complex conjugate of matrix A::

If A=$\begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix}$; $A^* = \begin{bmatrix} 1 & -6i \\ -3i & 2-4i \end{bmatrix}$;

$A^\intercal$ => transpose of matrix A

$A^\intercal = \begin{bmatrix} 1 & 3i \\ 6i & 2+4i \end{bmatrix}$

$A^\dagger$ -- Hermitian Conjugate (adjoint) of matrix A

If A= $\begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix}$, $A^\dagger = \begin{bmatrix} 1 & -3i \\ -6i & 2-4i \end{bmatrix}$

Note: $A^\dagger = (A^*)^T$

Fundamentals of Quantum Information ---Born Rule (measurement)

- Quantum state
  - $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$, where the coefficients are complex numbers
- The Born rule states, the probability of measuring $|0\rangle$ or $|1\rangle$ is the absolute value of α or β
- $\alpha^2 + \beta^2 = 1$ for the quantum state to be normalized
- Measurement operators, are matrices, ex.

$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, Z-basis , $|0\rangle$, $|1\rangle$

$$m0 = |0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$m1 = |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Fundamentals of Quantum Information
---Born Rule (measurement) (2)

- $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$

  $P = \langle \varphi|m_0|\varphi\rangle = \langle \varphi|0\rangle\langle 0|\varphi\rangle = |\langle 0|\varphi\rangle|^2 = |\langle 0|(\alpha|0\rangle + \beta|1\rangle)|^2$

  $= |\alpha|^2$

  Note: $\langle 0|0\rangle = 1, \langle 0|1\rangle = 0$

  Prob= $P_0 + P_1 = \langle\varphi|m_0|\varphi\rangle + \langle\varphi|m_1|\varphi\rangle = \langle\varphi|m_0 + m_1|\varphi\rangle$

  $= 1$; $m_0 + m_1 = I$

- Normalization; $|\varphi\rangle \xrightarrow[0]{\quad} |\Phi\rangle$; $|\Phi\rangle = m_0|\varphi\rangle = |0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle)$

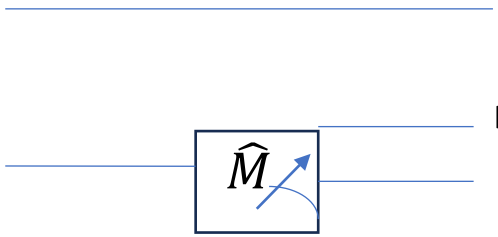
  $= \alpha|0\rangle$; $\alpha \neq 1$

Instead, $|\Phi\rangle = \frac{1}{\sqrt{p0}} m_0|\varphi\rangle = \frac{1}{\sqrt{|\alpha|2}} |0\rangle\langle 0|\varphi\rangle = \frac{\alpha}{|\alpha|}|0\rangle$

Length = 1

Generalized Born Rule— (Measurement)

$|\psi>$



$|\psi_j>$

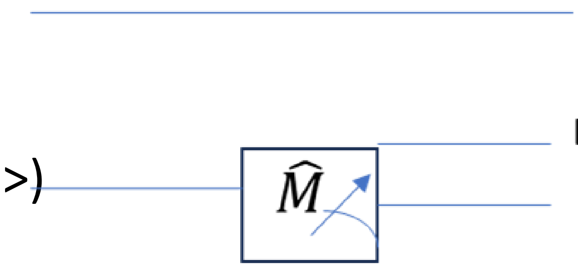
m = λ j

$|\lambda j>$ eigen vector $\hat{M}\,|\lambda_j> = \lambda_j\,|\lambda_j>$
with Probability $|\alpha|^2$

Two qubit state: $|\psi> = \sum_j \alpha_j |\psi_j>|\lambda_j>$; where $|\lambda_j>$ form the basis for measurements,

$|\psi_j>$ are normalized, not orthogonal
and
$\sum_j |\alpha_j|^2 = 1$

Two qubit state measurement – Example



$|\psi_j>$

$|\psi>$

$\frac{1}{\sqrt{2}}(|0+> + |1->)$

m = λ j

$|\lambda j>$ eigen vector $\hat{M}\,|\lambda_j> = \lambda_j\,|\lambda_j>$

with Probability $|\alpha|^2$

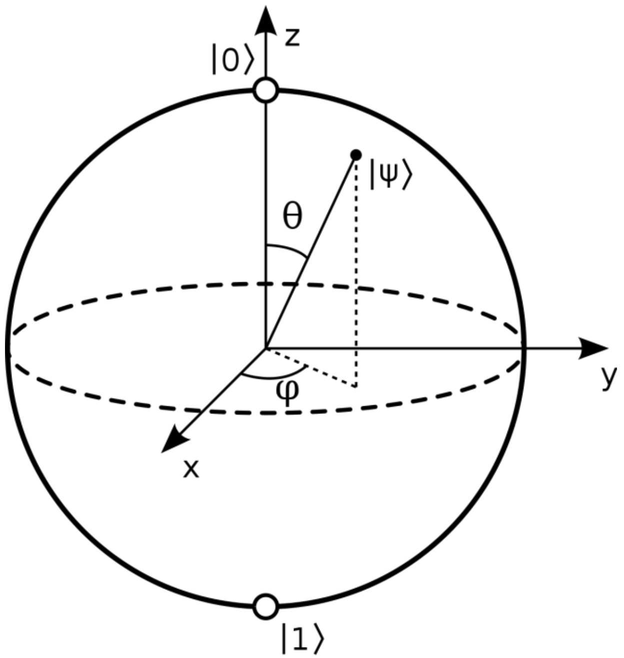
$\frac{1}{\sqrt{2}}(|0+> + |1->) = \frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(|00> + |01>) + \frac{1}{\sqrt{2}}(|10> + |11>)]$; where $|\pm> = \frac{1}{\sqrt{2}}(|0> \pm |1>)$

$= \frac{1}{\sqrt{2}}[\frac{1}{\sqrt{2}}(|00> + |10>) + \frac{1}{\sqrt{2}}(|01> + |11>)]$; Note: Rearrange the terms.

$= \frac{1}{\sqrt{2}}(|+0> + |-1>)$

p= ½, |0>, p=1/2, |1>

|+>, m = +1, |->, m = -1 :: X-Basis M+ = |+><+|, M_ =|-><-|



Entanglement(Bell State)

$|\Phi+\rangle = 1/\sqrt{(2)}\,(|0_A 0_B\rangle + |1_A 1_B\rangle)$

$= 1/\sqrt{2}(|+_A +_B\rangle + |-_A -_B\rangle)$

Superposition

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

If $|\alpha|2 = |\beta|^2 = 1/2$; 50% ; $|0\rangle$, 50%; $|1\rangle$

Inner product

$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$

$= \begin{pmatrix}\alpha\\\beta\end{pmatrix}$

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\quad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

Qubit States

Superposition

$|\Psi\rangle = |000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle$

3-bits

Entanglement

- <u>2-Qubit Operation</u>
  - Beam with multiple frequencies brings about interaction of electron, states.
  - Two ions in proximity,
  - Entanglement:

Only in Quantum Mechanics, Quantum thing can Entangle things. Entanglement is a pure quantum phenomenon, independent of distance. Strong corrections => EPR pair: Bell inequalities.

- Entanglement: (Spooky action at distance—Einstein)

  1. Flip two coins, two coins' outcome are not correlated. (50% tails, 50% head)

  2. Entanglement—correlated

Coin 1	Coin 2	
H	T	
H	H	
H	H	
T	T	
T	H	
H	T	

Spin 1	Spin 2	
↑	↑	
↓	↓	
↑	↑	
↓	↓	
↑	↑	
↓	↓	

$$|\varphi\rangle = (1/\sqrt{2})(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Entanglement Notes:

- Notes: 1935: Einstein, Podolsky and Rosen (EPR)—Quantum physics is an “Incomplete” . The predictions of Quantum physics are “Probabilistic”. EPR presented a scenario that the quantum particles, electron and photons carry physical properties or attributes not included in quantum theory—uncertainties. “Hidden variables”
- 1964: John Bell demonstrated Entangled pairs no Hidden Variable.

Quantum Bits– Basic

Review of Basic Concepts of Quantum Bits

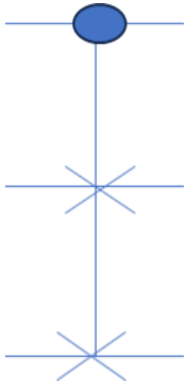
Quantum Bits requirements

(Quantum Bits basic knowledge for new audiences in the Quantum field.)

Quantum Logical Gates

Source: Wikipedia

Fredkin Gate



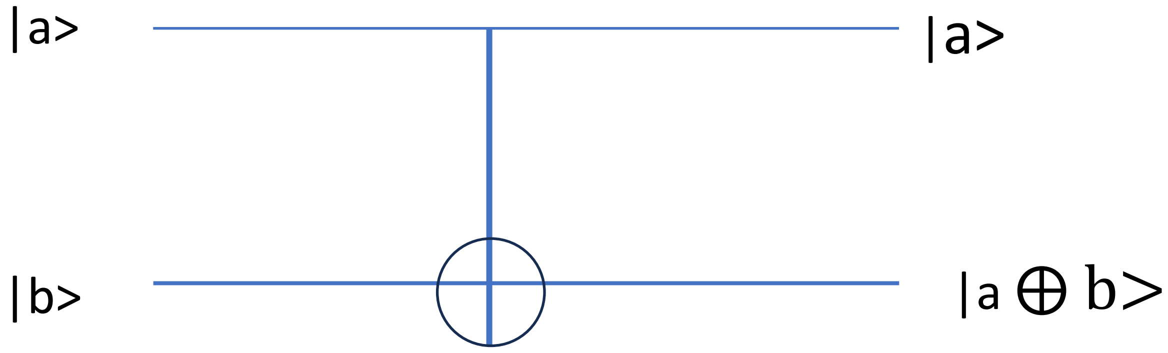
-Universal gate

-Reversible

Operator	Gate(s)		Matrix
Pauli-X (X)	X	⊕	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)	Y		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)	Z		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)	H		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)	S		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)	T		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)	Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

The CNOT Gate

|00> → |00>
|01> → |01>
|10> → |11>
|11> → |10>



	|00>	|01>	|01>	|11>
|00>	1	0	0	0
|01>	0	1	0	0
|01>	0	0	0	1
|11>	0	0	1	0

Quantum Bits -- Requirements

- Di Vincenzo Criteria
- Two level System, Bloch Sphere
- Qubit Gates
- Relaxation and Dephasing

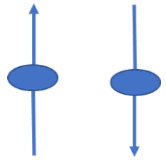
The DiVincenzo Criteria for Quantum Computer

- Requirements for the physical implementation of Quantum Computer
  - D1: Scalable Qubits
  - D2: Initialization
  - D3: Measurement
  - D4: Universal Gate Set
  - D5: Coherence
- Requirements for routing Quantum Information
  - D6: Interconversion
  - D7: Communication

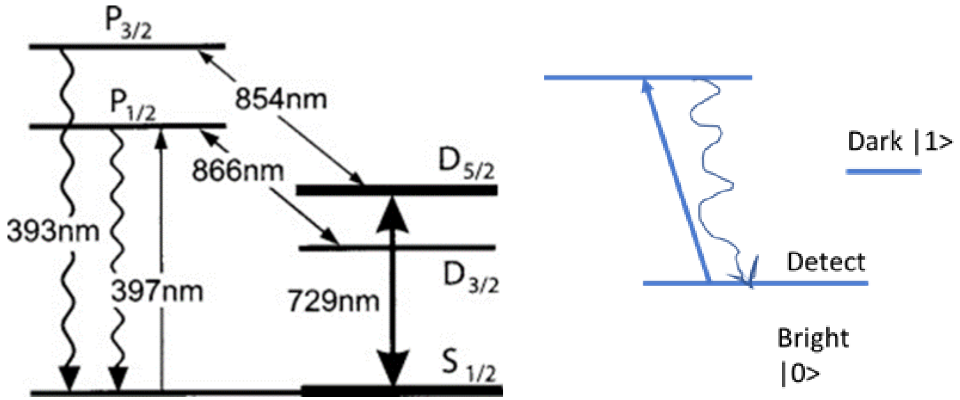
Two level system: Two level Quantum System

Photon Polarization

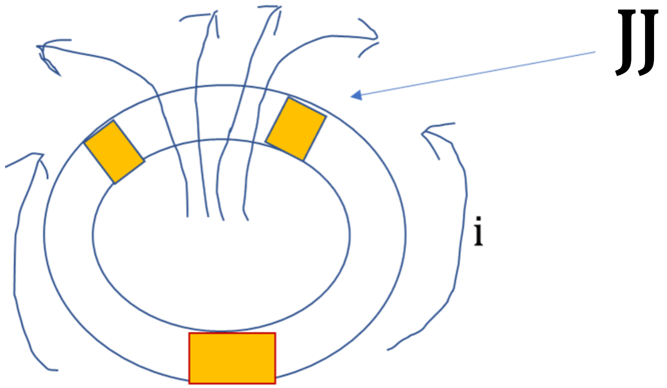
Spin ½--Spin up, Spin down



Trapped Ion (Ca^+)



Superconductor ring with Barrier



(Current flows RIGHT or LEFT)

Qubit state:

n, 2-level System vs (One 2^n total system)

$|1\rangle \otimes |2\rangle \otimes$ ------------- $\otimes |n\rangle$

$10110 = |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |1\rangle \otimes |0\rangle$

N, 2-level System vs. (One 2^n total system)- (2)

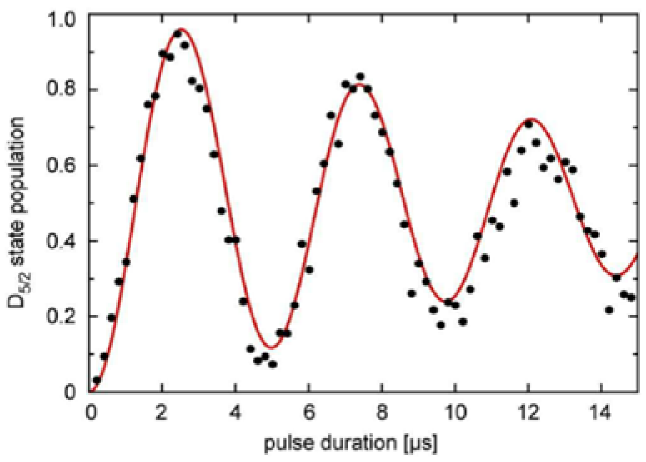
- Scalability and cost of he system:

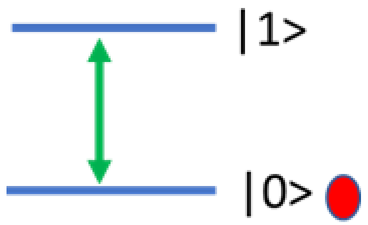
a. How large of a system can you build?
b. How easily can you add more qubits processors?
c. Measurement is to determine probability, many times (repeated)

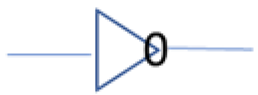
1-Qubit:

$$|0> \otimes |1> = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = 01$$

Rabi Oscillation: Two level system, The Ground state to Excited State, Periodic exchange energy, 2π frequency.







Control to π/2 ; Superposition State →1/√2 (|0> + |1>)

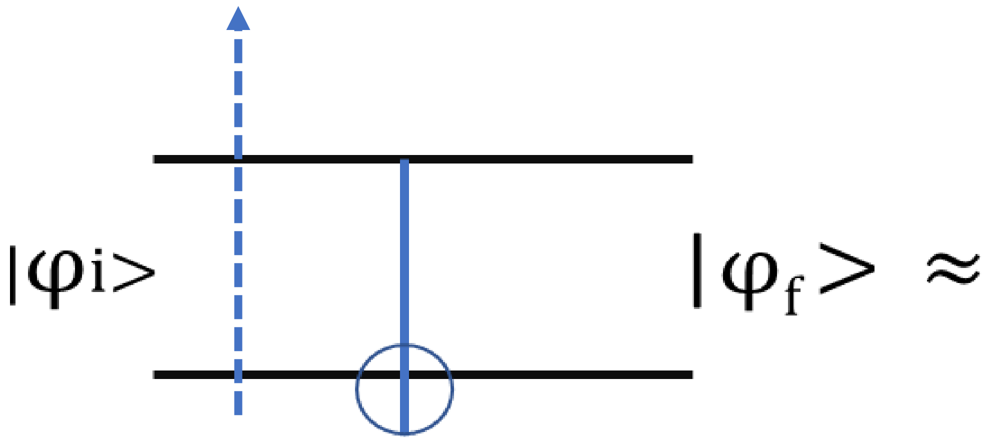
Entanglement –
(Spooky action at distance- Einstein)

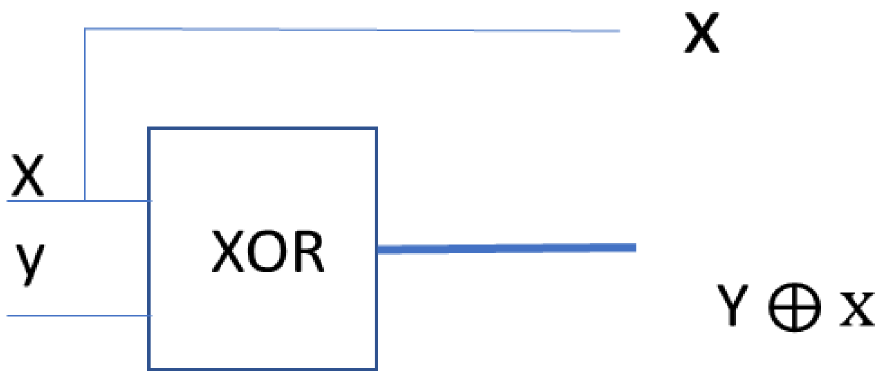
- 2-Qubit Operation
  - Beam with multiple frequencies brings about interaction of electron, states.
  - Two ions in proximity, Entangled pair → a Quantum State
  - Entanglement:

Only in Quantum Mechanics, Quantum thing can Entangle things. Entanglement is a pure quantum phenomenon, independent of distance. Strong corrections => EPR pair: Bell inequalities.

$|\varphi i> = (|0> + |1>)_1 \; |0>_2$

$|\varphi_f> = |0>_1 |0>_2 + |1>_1 |0>_2$ ➔ $|\varphi_f> \; = |0>_1 |0>_2 + |1>_1 |1>_2$





Particle 1: $|u_1\rangle, |u_2\rangle$

Particle 2: $|v_1\rangle, |v_2\rangle$

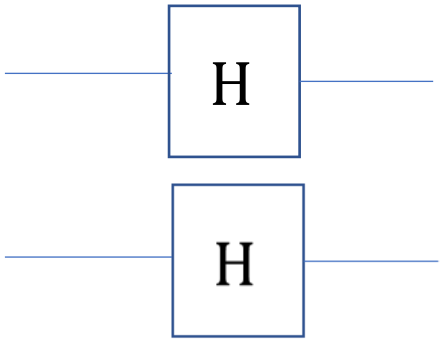
$|u1\rangle \otimes |v1\rangle + |u2\rangle \otimes |v2\rangle \neq (\ldots\ldots) \otimes (\ldots\ldots.)$

A state of two particles is said to be Entangled, if it cannot be factorized form $(\ldots.) \otimes (\ldots.)$.

Spin electrons: $|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle$ ← Entangle State of the pair of electrons.

Not Entangled

- $|0>_A$
- $|0>_B$

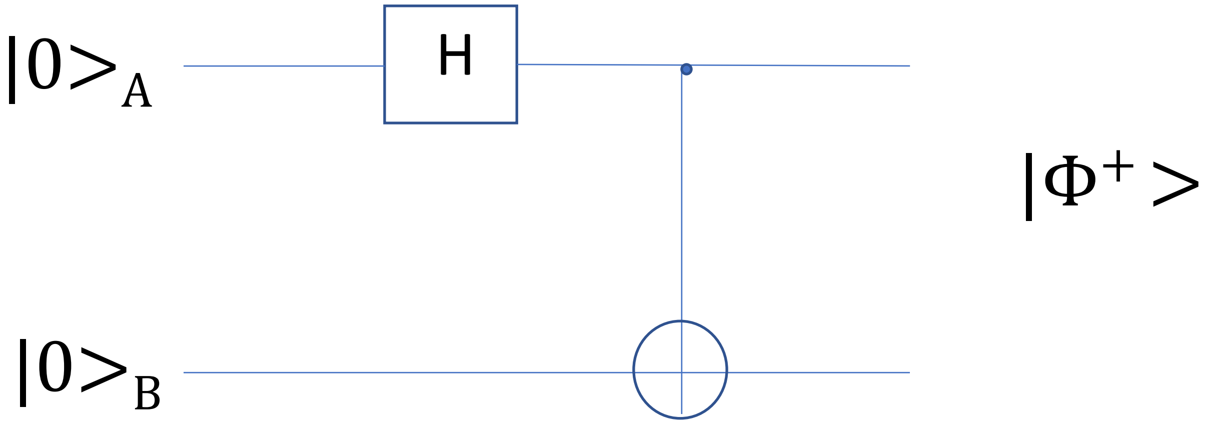


$|\psi>$

$$|\psi> = H|0>_A \otimes H|0>_B$$
$$=1/\sqrt{2}\,(|0_A0_B>+|0_A1_B>+|1_A0_B>+|1_A1_B>)$$
$$=1/\sqrt{2}\ (|0>+|1>)_A \otimes\ +1/\sqrt{2}\ (|0> +|1>)_B$$
$$= |+> \otimes |+>$$

Entangled State(Bell State)

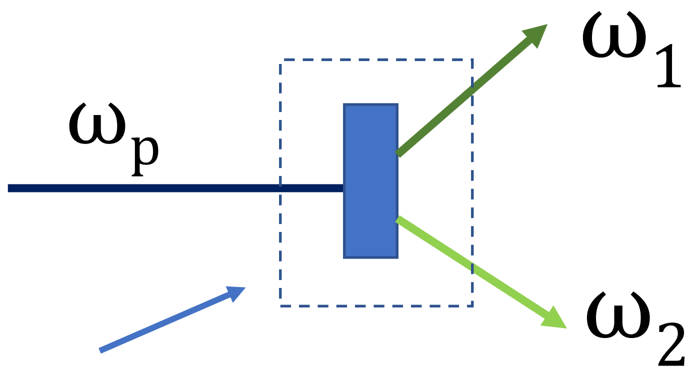
$$|\Phi^+> = 1/\sqrt{2}\ (|0_A 0_B> + |1_A 1_B>)$$
$$= 1/\sqrt{2}(|+_A +_B> + |-_A -_B>)$$



How to create Quantum Entanglement?

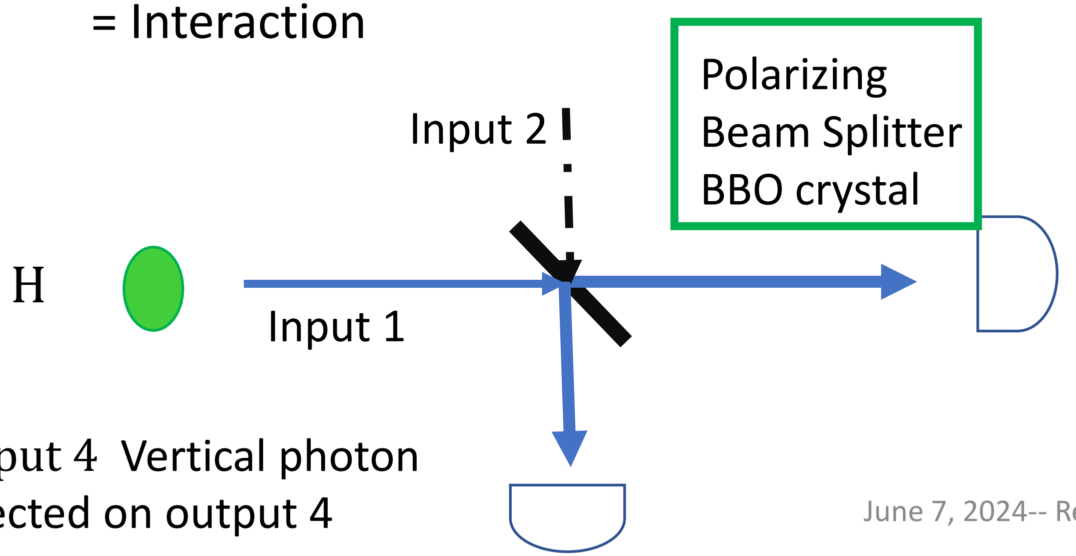
Spontaneous Parametric Down Conversion (SPDC)

Generated two Photons are correlated—polarization entangled



$$\hbar\omega_p = \hbar\omega_1 + \hbar\omega_2$$

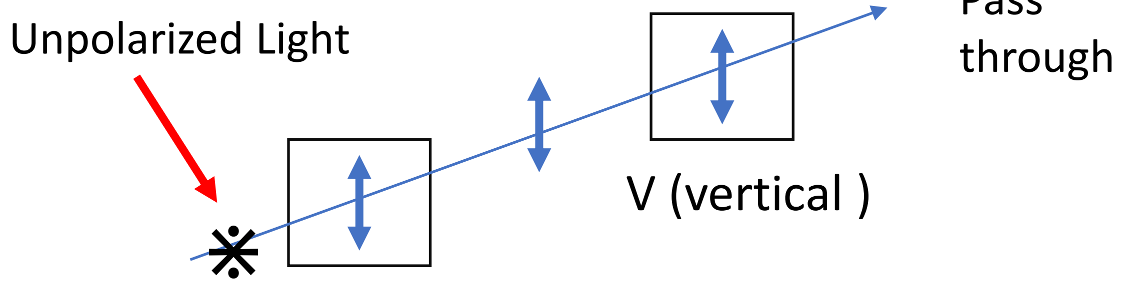
[Non-linear Crystal]
= Interaction

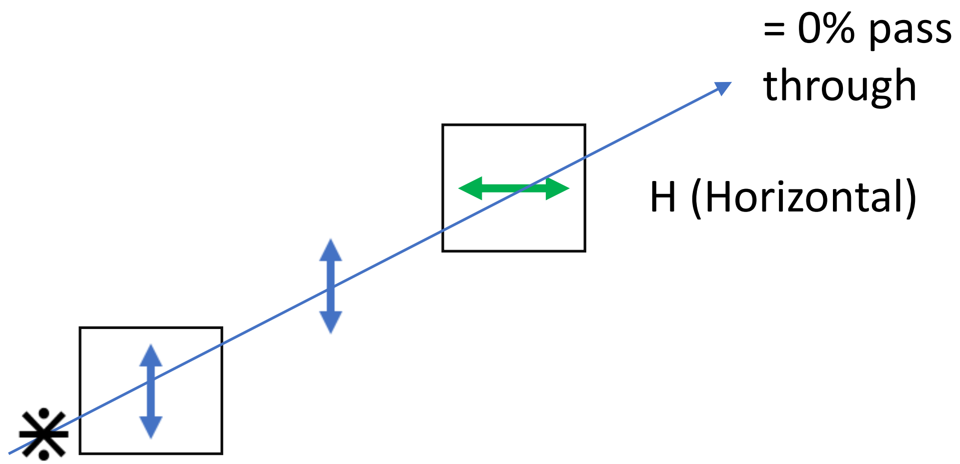


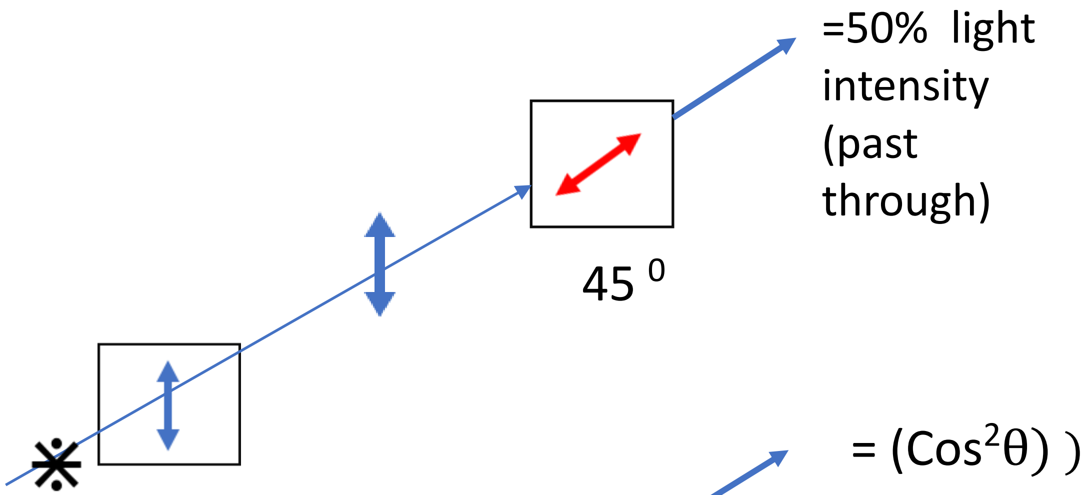
Output 3 Horizontal photon is transmitted, detected on Output 3

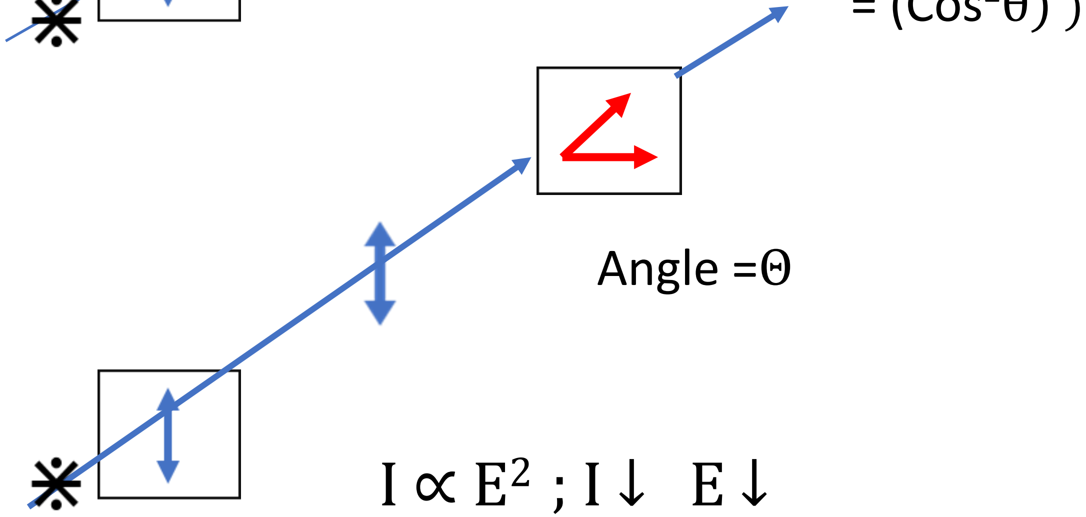
Output 4 Vertical photon reflected on output 4

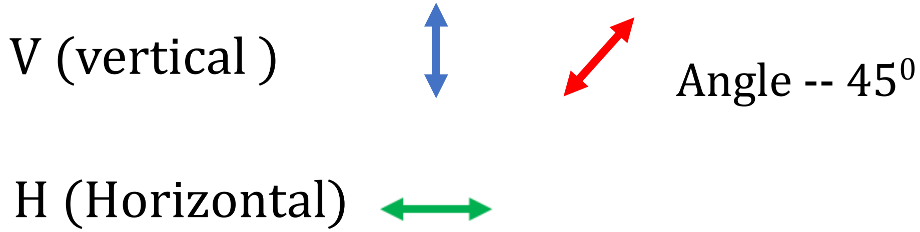
Polarization











$I \propto E^2$; I ↓ E ↓

Energy, E= $\hbar$ f

frequency = does not change

Intensity = reduced

H  $|\chi\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ $\quad |\langle x|y\rangle|^2 = [1\ 0]\begin{bmatrix}0\\1\end{bmatrix} = 0;$ (Orthogonal)

0% pass through

V  $|y\rangle = \begin{bmatrix}0\\1\end{bmatrix}$ $\quad |\langle x|x\rangle|^2 = [1\ 0]\begin{bmatrix}1\\0\end{bmatrix} = 1^2 = 1$

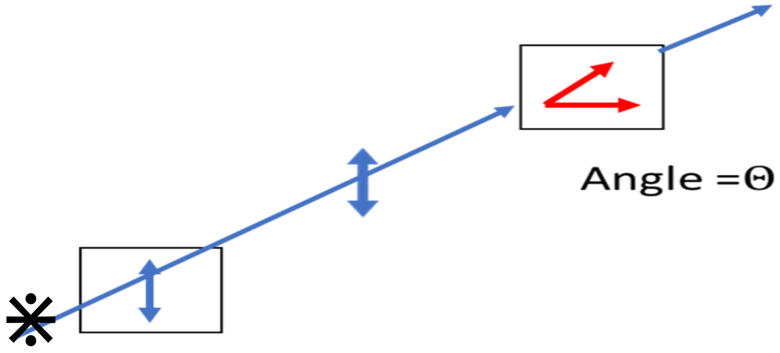
(H → H) (100% Pass through)

$|\langle y|y\rangle|^2 = [\ 0\ 1]\begin{bmatrix}0\\1\end{bmatrix} = 1^2 = 1$

Probability amplitude: $\langle\chi|y\rangle|^2 = \langle\chi|y\rangle\langle y|\chi\rangle$

$= (\mathrm{Cos}^2\theta)\ )$

= 50% pass through



$\mathrm{Cos}^2\Theta = ½$

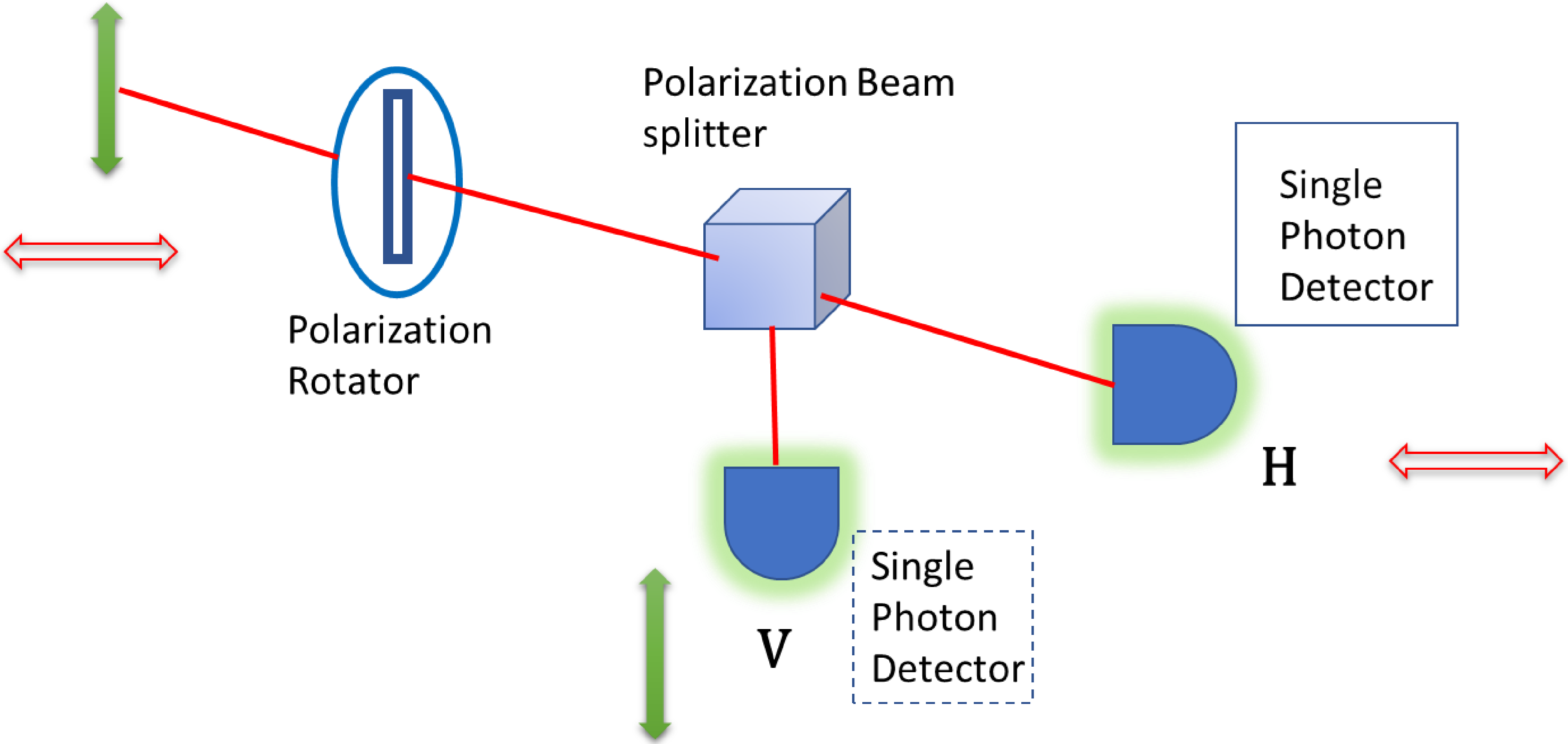
$\theta = 45^{\,0}$

$|\chi>$ $\leftrightarrow$

$|y>$ $\updownarrow$

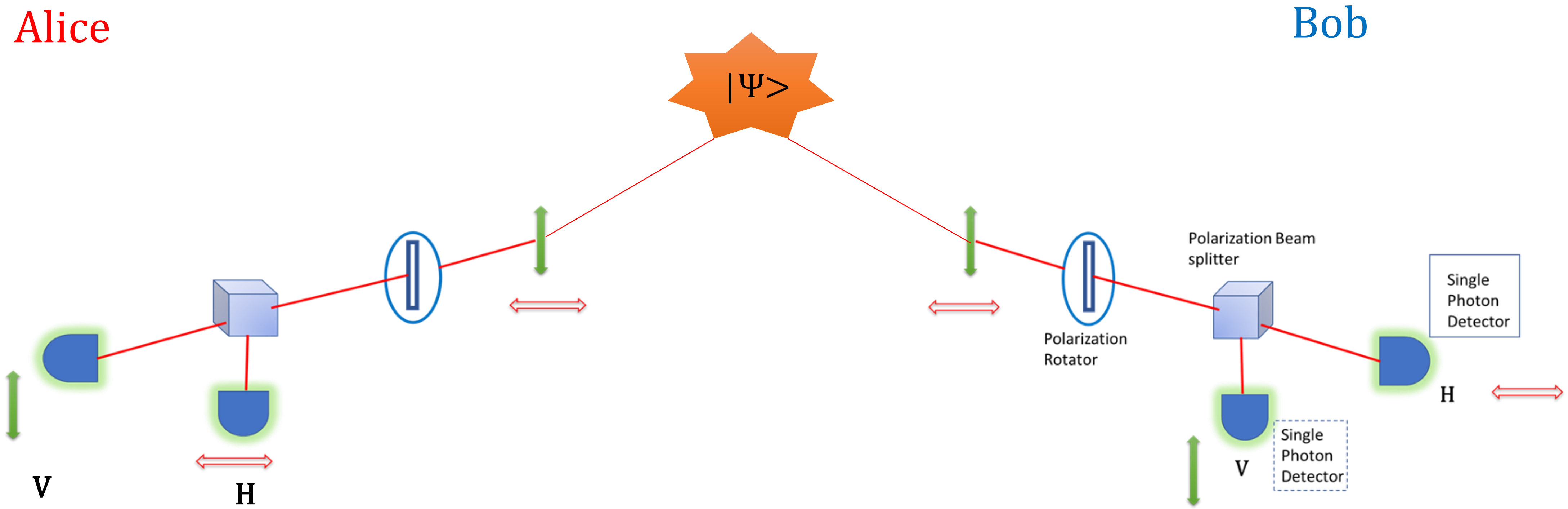
$$|\nearrow> = \frac{(\,|\chi> + |y>\,)}{\sqrt{(2)}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|\searrow> = \frac{(\,|\chi> - |y>\,)}{\sqrt{(2)}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$



Polarization Beam splitter
Single Photon Detector
Polarization Rotator
H
Single Photon Detector
V

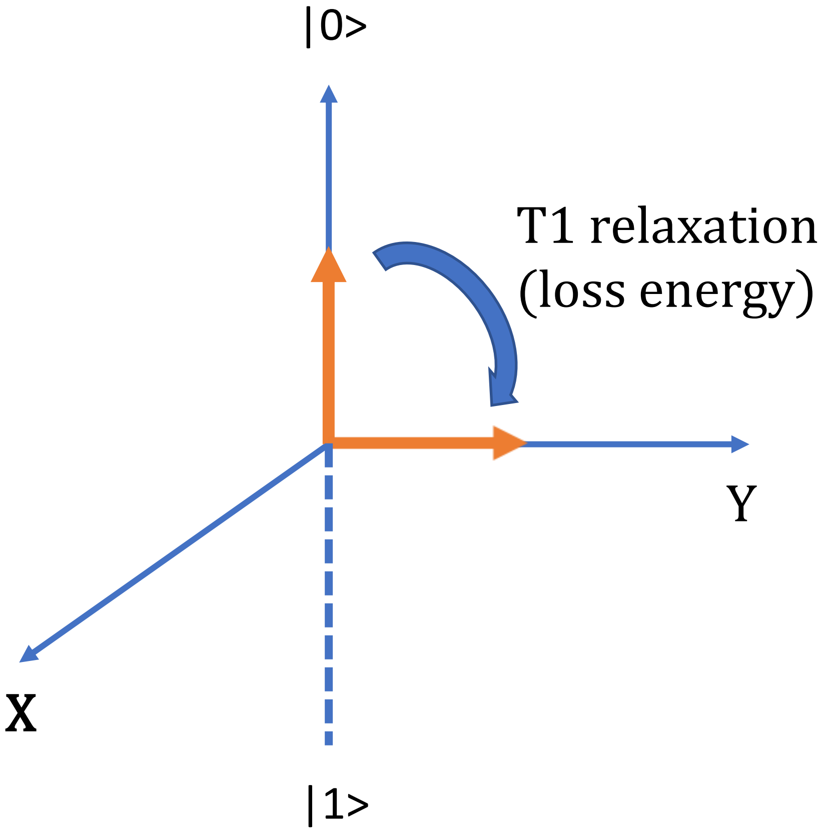
$$|\Psi> = (1/\sqrt{2})(|H_A V_B> + |V_A H_B>)$$

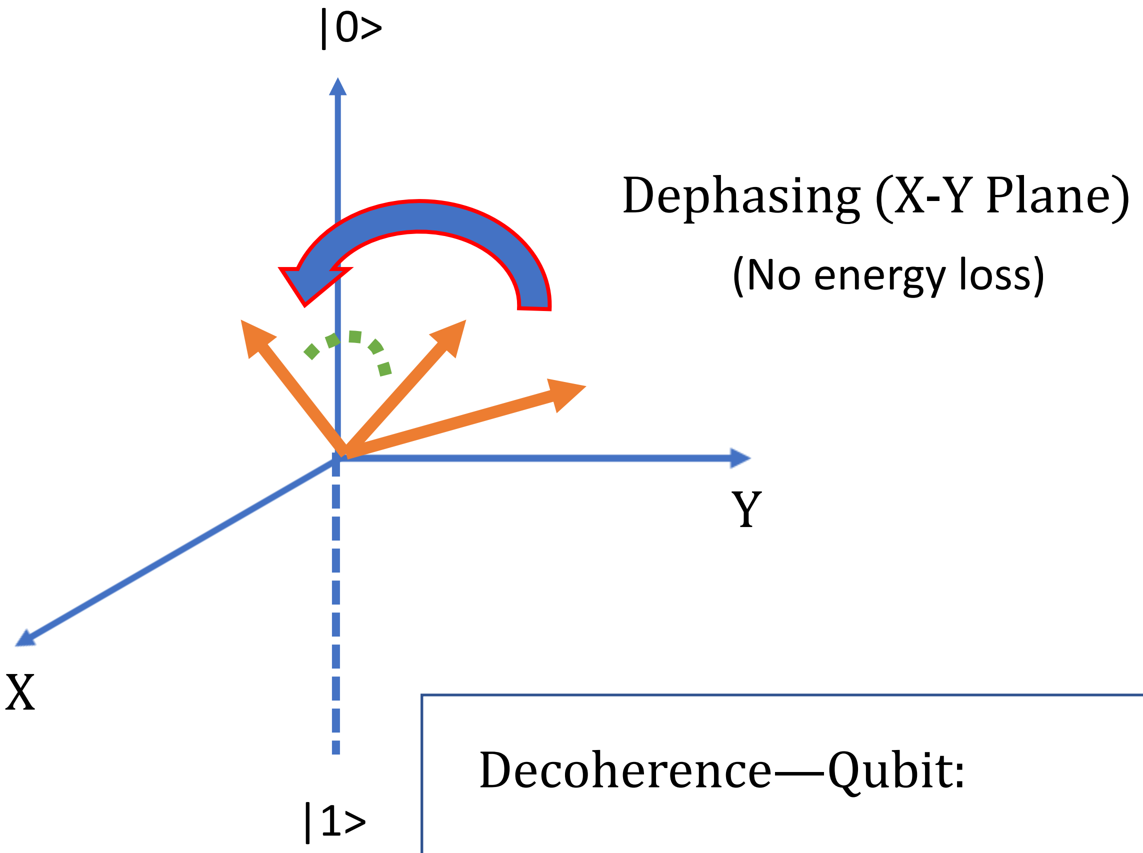


Qubit Coherence and Gate time

- Qubit Errors
  - Energy Relaxation, T1
  - Decoherence, T2
- Clock speed at which Qubit operations can be performed
- Gate Fidelity
- Long coherence time does not translate to more operations per gate time
- Long-lived Qubit modalities, slow Gate time.

Decoherence





$$T_2 < 2T_1, \quad T2 << T_1$$

Decoherence—Qubit:

0 Environment (Lab. equipment, Magnetic fields)

0 Unknown (Fluctuating spins, in environment)

Gate Fidelity

- Gate Fidelity equals to 1– Test qubit's operation is identical to the theoretically predicted output state, 100% of the time.
- How well a gate operation works?
- Comparing of actual Implemented gate operation and ideal, theoretically calculated gate
- Process tomography
  - Process tomograph- Characteristic gate operation
    - Sensitive to state preparation and measurements errors
    - Randomized benchmarking– The random gates are first characterized by themselves to assess a baseline of errors including SPAM errors. Then the same measurement is performed with the desired gate operation. The results compared and the additional error is attributed to the addition of the desired gate operation

Gate Fidelity-- Process tomography

- Process tomography
  - Process tomograph- Characteristic gate operation
    - Sensitive to state preparation and measurements errors
    - Randomized benchmarking– The random gates are first characterized by themselves to assess a baseline of errors including SPAM errors. Then the same measurement is performed with the desired gate operation. The results compared and the additional error is attributed to the addition of the desired gate operation
  - SPAM means State Preparation and Measurement, span errors: a leading metric that quantum computer providers are using to measure the accuracy and reliability of their devices.

More requirements—Qubit:

- Gates must be stable (precise, system errors)
  - Calibration, cross-talk,
  - Hardware accuracy (limitation)
    - Pulse timing, phase noise, etc.)
- Gate must fast
  - $>$ 10000 faster than coherence time, Parallelization
- Measurements
  - $a|0> + b|1>$ → measure "0" , $||a||^2$, "1" , $||b||^2$ (probability, repeat many times)
  - Measurement with limitation
    - Repeated calculation
    - Quantum Fan-Out (Error Correction)
- De-coherence: T1, T2

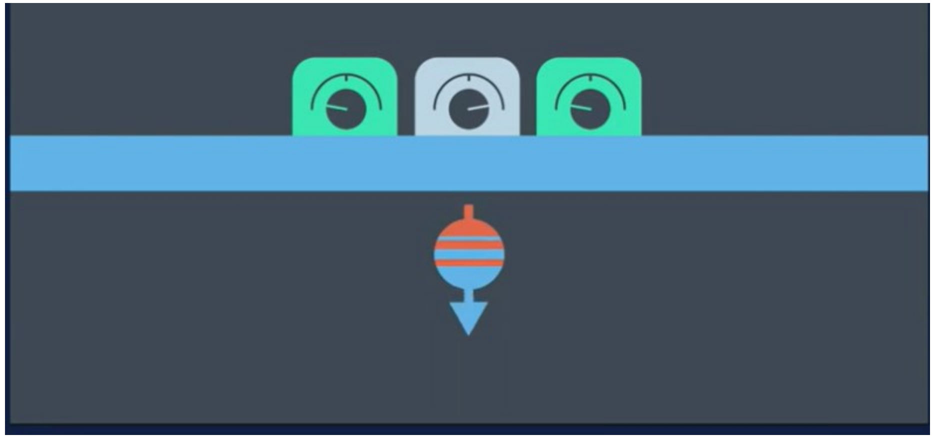
Qubit Modalities
(Electron and Nuclear Spins)

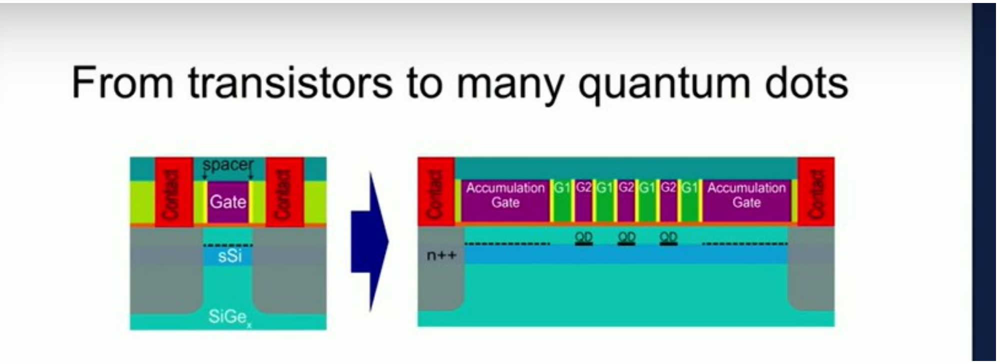
- Spin Qubits
  - Silicon-based spin Qubit technology
  - Compatible CMOS Process technology (Traditional CMOS process), by leverage the CMOS know-how from IC chip industry
  - Quantum dots placed between the source and drain on CMOS silicon substrate, SOI (silicon-on-insulator)
  - Control circuits: Microwave cavity and measured via gate-based dispersive readout
  - For large-scale Universal Quantum computation– Quantum Error Correction (ECC)

Source: Nature 479,345 (2011)

Electron Spin Qubits (2)

- Single spin qubits encode quantum information with in the intrinsic spins of electrons, and perform computation by manipulating those spins. Instead of quantum dots
- Spin qubit, so far have been implemented in semiconductors, such as Gallium Arsenide, Silicon, and Germanium.
- Spin qubits have also been implemented in Graphene.





Source: QuTech Academy

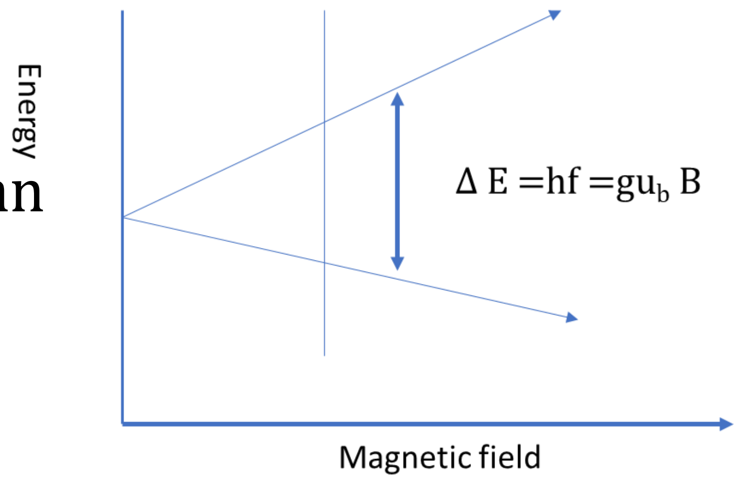
Qubit Modalities(3)– Electron Spin

- SiGe Quantum Dots
  - Electron Spin is trapped in a Quantum Dot, a small region of semiconductor material where a single electron can be trapped

- Advantages of Quantum Dots
  - Leverage Silicon Fabrication Technology
  - Small in area
  - Controlled by Gate voltages
- Challenges: Multiple gates, 3D-integration technologies required

Qubit Modalities(4)– Electron Spin

- Magnetic field split the spin states, spin-up and spin-down
- Magnetic field split the spin states separated by the Zeeman energy
- Low energy state is spin ↑ to spin ↓
- |↑> → spin up ("0"),
- |↓> → spin down ("1")
- Physical qubit -> |Ψ> = α|↑> + β|↓> =α|0> + β|1>



Where g , for B=1 T, Ez = 116uEV

f = 28GHz @ 1T

|↑> will acquire a phase relative to |↓> with a frequence $\mathcal{F}$, call Larmor frequency

Nitrogen Vacancies

- Diamond with Nitrogen Vacancy
  - NV-Centers, a nitrogen atom is injected into a diamond lattice causing a carbon vacancy to atom
  - Operated at room temperature
  - Long Coherence time => have the ability to communicate Quantum information, and have the ability to interconnect stationary and flying qubits.

Qubit Modalities: Trapped Ion Qubits

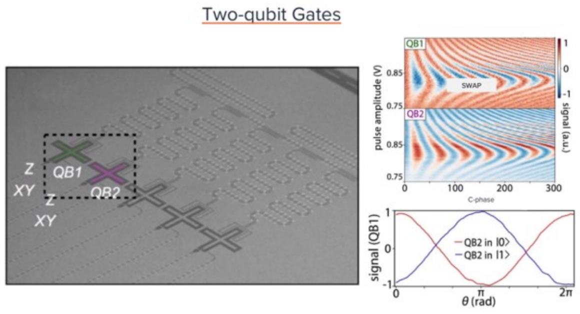
- Used as atomic clocks for decades, system are stable and well characterized
- Satisfy DiVincenzo Criteria
- Ion’s charged, it can be trapped, or hold in place using oscillatory electromagnetic fields
- Advantages:
    - Leverage of current fab technology, silicon based technology
    - Control and Readout circuits can be integrated with CMOS process

Qubit Modalities: Superconducting Qubits

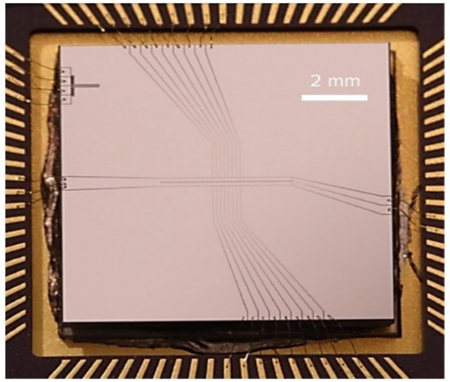
- Manufactures artificial atoms
- Superconducting Qubits are electrical circuits
- Superconducting Qubits are Non-linear oscillator built from Capacitors and Inductors—Josephson Junction
- Gate fast/Manufactured on silicon CMOS process
- Challenges:
  - Low temperature (milli Kelvin Temperature)
  - Integration of control and readout that maintain Qubit coherence at low temperature
  - 3D integration technology is required

Qubits Technology Summary:

- Superconducting Qubits
- Trapped Ion
- Topologic Qubits
- NV centers
- Photonic
- Silicon

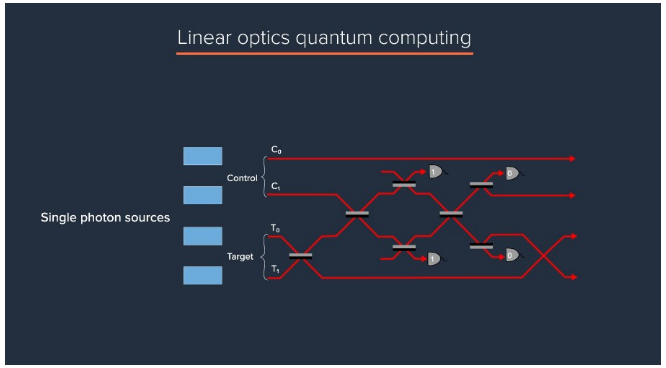


Superconductor

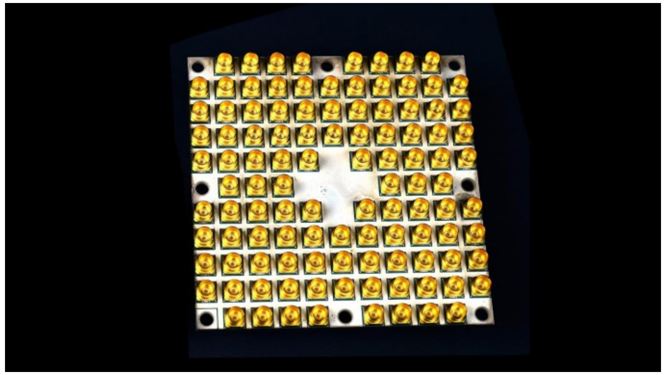


Surface-Electrode Trap Chip

Trapped Ion



Photonic



Silicon

Sources: Intel and MIT-Lincoln Lab.

Qubits Technology	Number of qubits	T1/T2 time(ns) Fidelity Readout time	Scalability	Advantages	Disadvantages
Superconductors	433	50us/100us 99.9% 5MHz	Possible, qubit size, and scale of integration CMOS compatibly	Well researched technology CMOS compatible process Conventional control equipment,	Low coherence time, fast gate Sensitivity to noise Low temperature(15 to 20mK)
Trapped Ions	53	> $1e^{14}$ (Years)/50s 99.0% $1.00e^{-4}$MHz	Difficult, High level of integration is difficult. CMOS compatibly	Good stability Long coherence time, slow gate operation 4K to 10K temperature Laser as control equipment	Too slow, slow quantum calculation
Photon	20		Yes, Silicon technology	High operating temperature CMOS technology, photons are using in telecom	High error rate, No possibility to store photons
Silicon(SOI, SIGe)		1000ms/0.4ms 99.6% 1MHz	Yes, Silicon technology	CMOS technology, Fast quantum gates,	
NV Centers		100ms/200ms 94% $2.0e^{-02}$ MHz		High temperature (4K) Long coherence time Used as memory	Complex scalability
Quasi particles (Anyon, fermions de Majorana)			May be, if it is semiconductor technology		

Superconductor Qubits

Why Superconductor?

Quantum Control Engineering—Control circuits, better Qubit

Quantum Engineering on the Read-out circuits– Cryogenic CMOS

Qubit Modalities: Superconducting Qubits

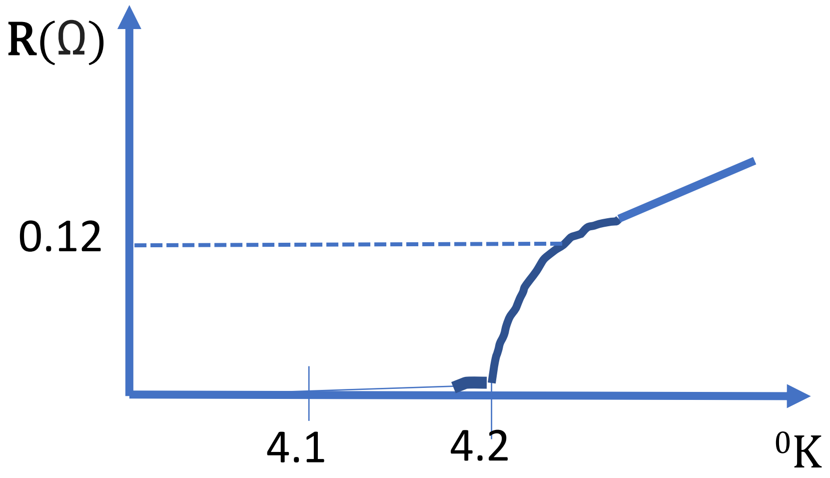
- Manufactures artificial atoms
- Superconducting Qubits are electrical circuits
- Superconducting Qubits are Non-linear oscillator built from Capacitors and Inductors—Josephson Junction
- Gate fast/Manufactured on silicon CMOS process
- Challenges:
  - Low temperature (milli Kelvin Temperature)
  - Integration of control and readout that maintain Qubit coherence at low temperature
  - 3D integration technology is required

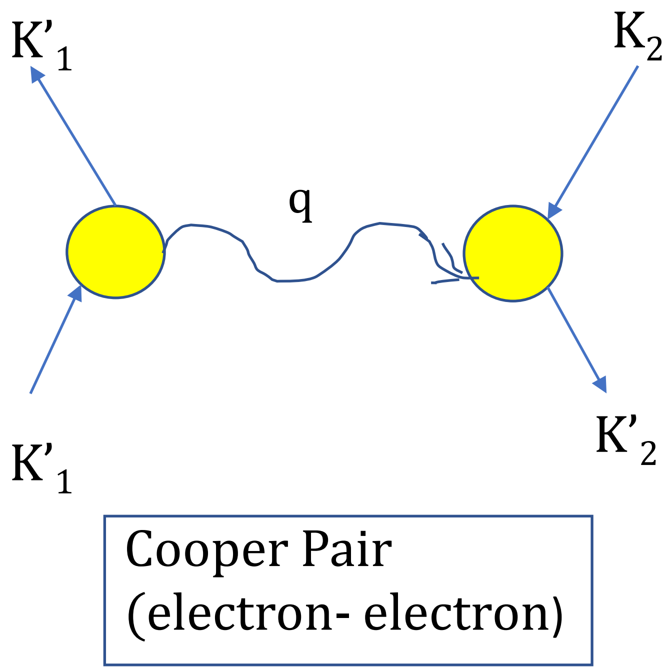
Superconducting Circuits

- A Quantum Computing modality
- Stores quantum information in superposition of charge and current
- Superconducting metal cools down below some temperature
  - ---→ (cool down) electron pair up into Cooper Pairing.
- Theory of Superconductivity, the BCS Theory, B-Bardeen, S-Schrieffer, C-Cooper
- BCS– John Bardeen, Leon Coopers, Robert Schrieffer, 1957 – 1972 Nobel Prize in Physics
- 1962– Josephson effect
- 1964– SQUID

Superconductivity[8]

- Superconductor basic electric characteristics





(Weakly bound)

Cooper-Pairs

- Superconductors cooled extremely low temperature to achieve Superconductivity.
- At this low temperature, electrons in the material tend to become bond to each other as Cooper-Pairs.
- Cooper Pairs are charge carriers of the superconducting system
- Cooper Pairs are the quasiparticles formed between two electrons with equal and opposite momenta, including the spins of electrons.
- Puali Exclusion Principle does not apply to Cooper-Pair.
- Cooper-Pairs are very stable, Cooper-Pairs highly resilient to disturbances (noise) caused by scatter event.

Superconducting Circuits–
Linear Harmonic Oscillator

- Inductor, L and a Capacitor, C
- LC Oscillator

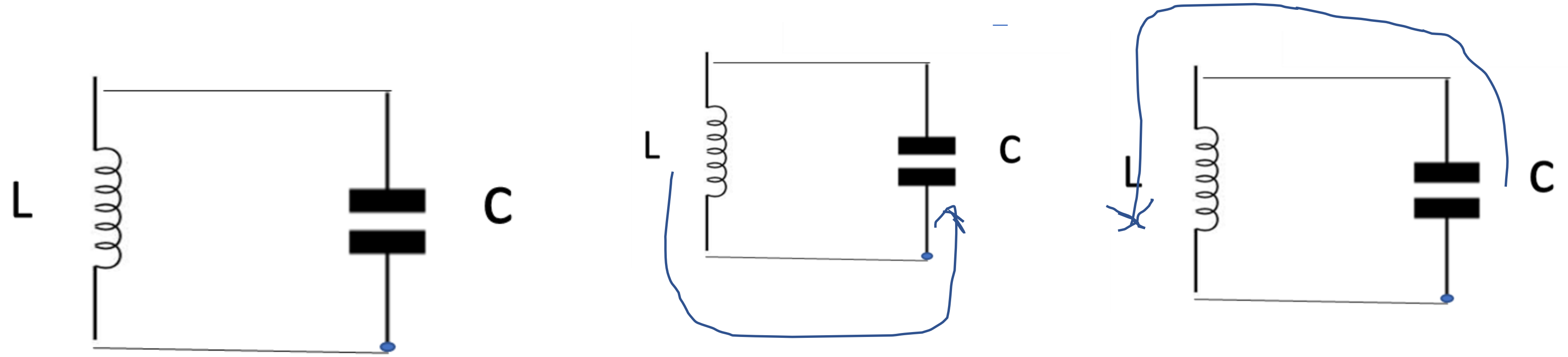
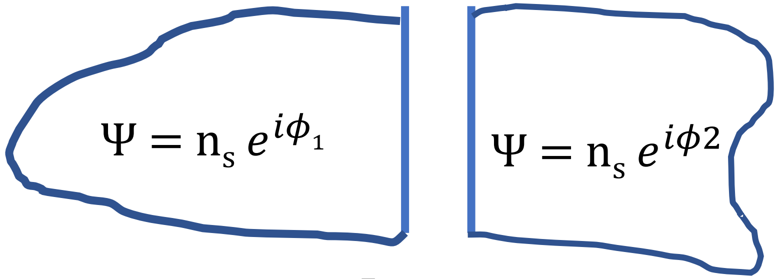


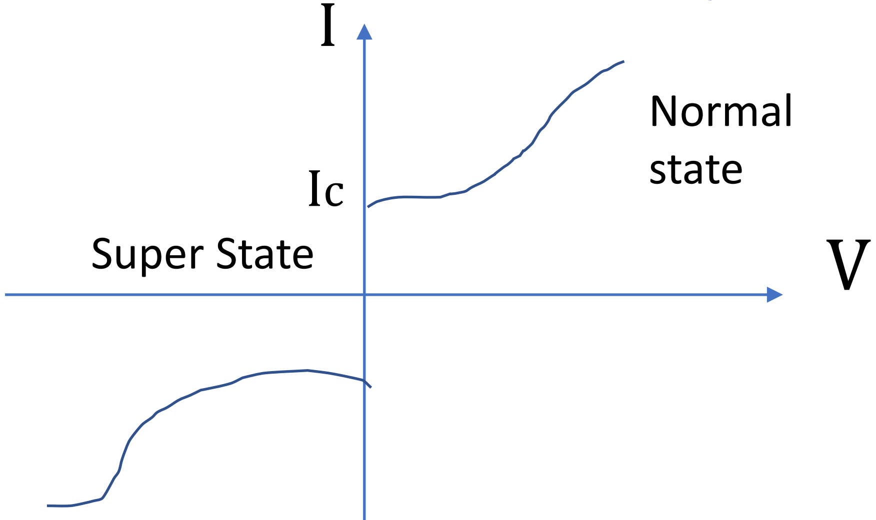
Fig. 1A

- Linear Harmonic Oscillator
- Frequency : $\mathcal{F}n = 1/2\pi(LC)^{1/2}$ ~ 5GHz
- Normal metal lose too much energy per oscillation

Josephson Junction-Basic concept

Nb
Superconductor



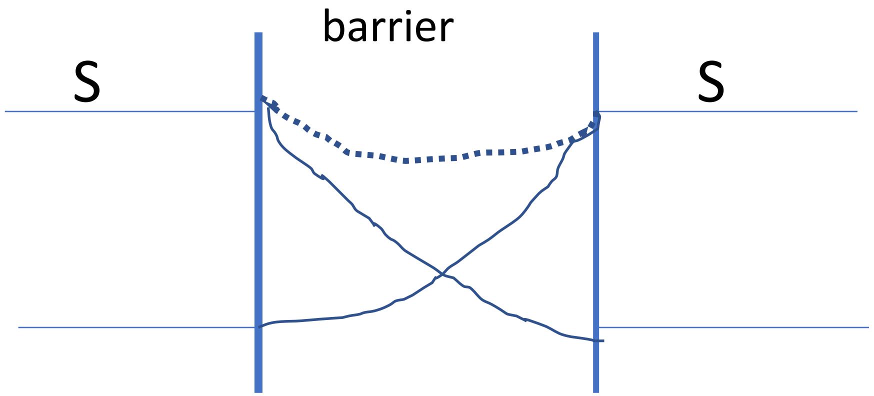


(Non-linear Transport)

Super Current between two superconductors,

$I_S = Ic \sin(\Delta\Phi)$

Phase drop across the junction, $\phi_1 - \phi_2 = \Delta\phi$

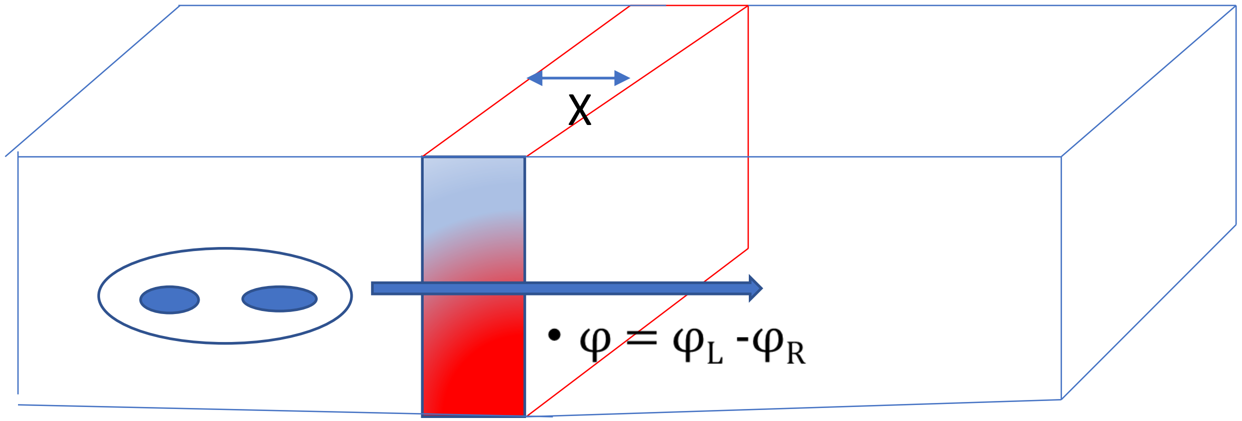


Left and Right tunneling – Cooper Pair

Josephson Junction(JJ): Nonlinear Inductors

- 1962 Brian Josephson
- 1973 Leo Esaki, Ivar Giaever, and Brian Josephson—Nobel Prize in Physics

(X-nm Thin Layer), X= 1 nm



- Current $I = I_c \sin\varphi$
- Voltage $V = (\Phi_0/2\pi)(\frac{d\varphi}{dt})$
- Inductance $V = L_j \frac{dI}{dt}$

- Superconductor current can tunnel through the barrier without lose energy

Physics of Josephson Junction:

- Josephson junction :: thin insulative gap (layer) ` 1nm (1 nano meter) between two superconductors. , AL/AlOX/Al. (Superconductor-Insulator-superconductor sandwich)
- Exhibits a non-liner I-V relationship (Current-Voltage) is the Key Property needed in designing Qubit.
- Josephson junction is a non-linear inductor.

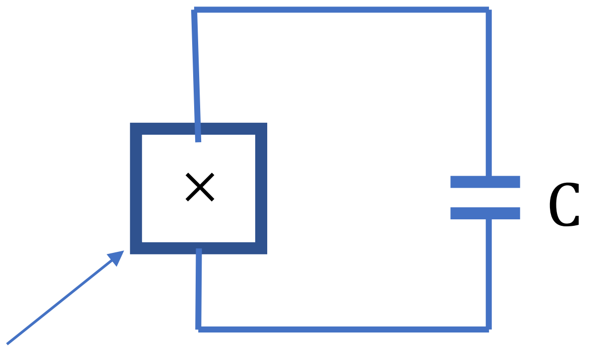
- By implementing (Engineering) Josephson Junction into different circuit elements create Qubit., individually to access the quantum states of the Qubit. Example: Transmon Qubit.

- Fabrication process of Superconducting Qubit (cQED) is the same as classical CMOS fabrication process.

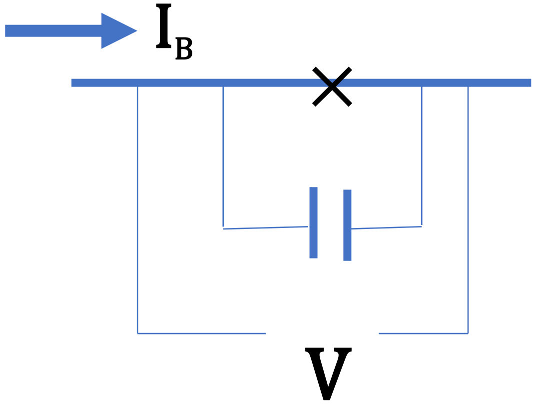
- Many challenges, and Opportunities to Fab these devices.

Nonlinearity--Superconducting Qubit

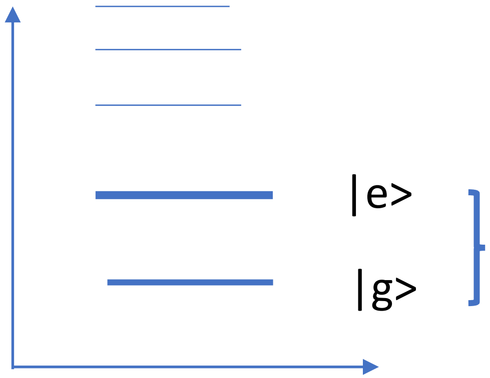
- Josephson Junction
  - Current depends on phase difference between two electrodes
  - Phase difference
  - Two superconductors electrodes separated by a thin barrier, oxide layer



JJ replaces
Inductance

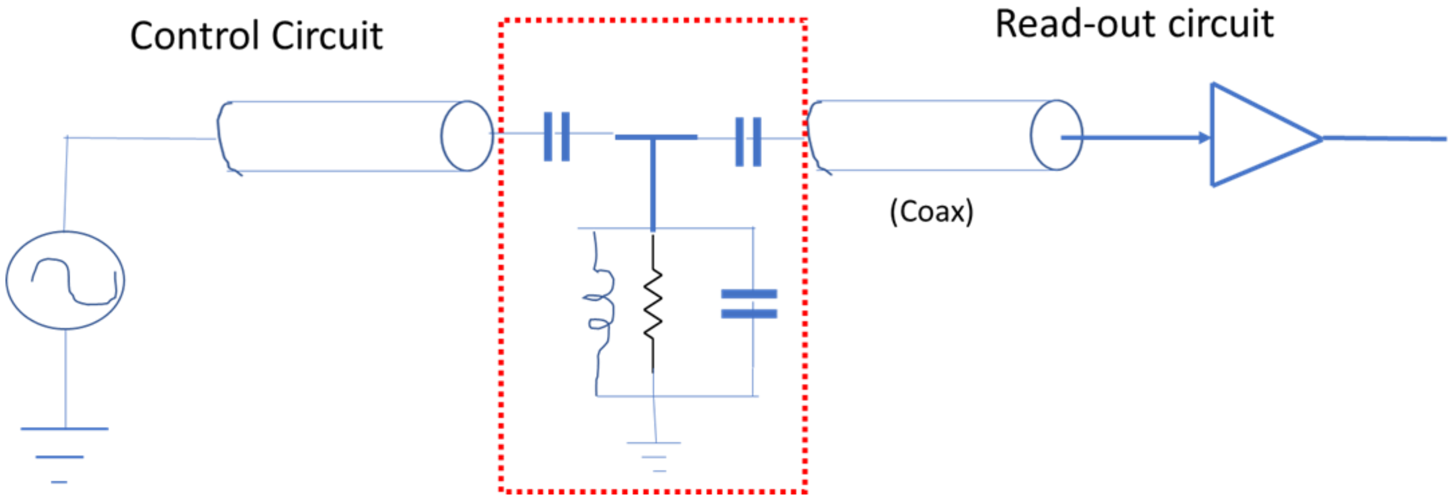


(Circuit diagram)



(Lowest
Level)
GND State

Quantum Circuit Operation



(Quantum Circuit)

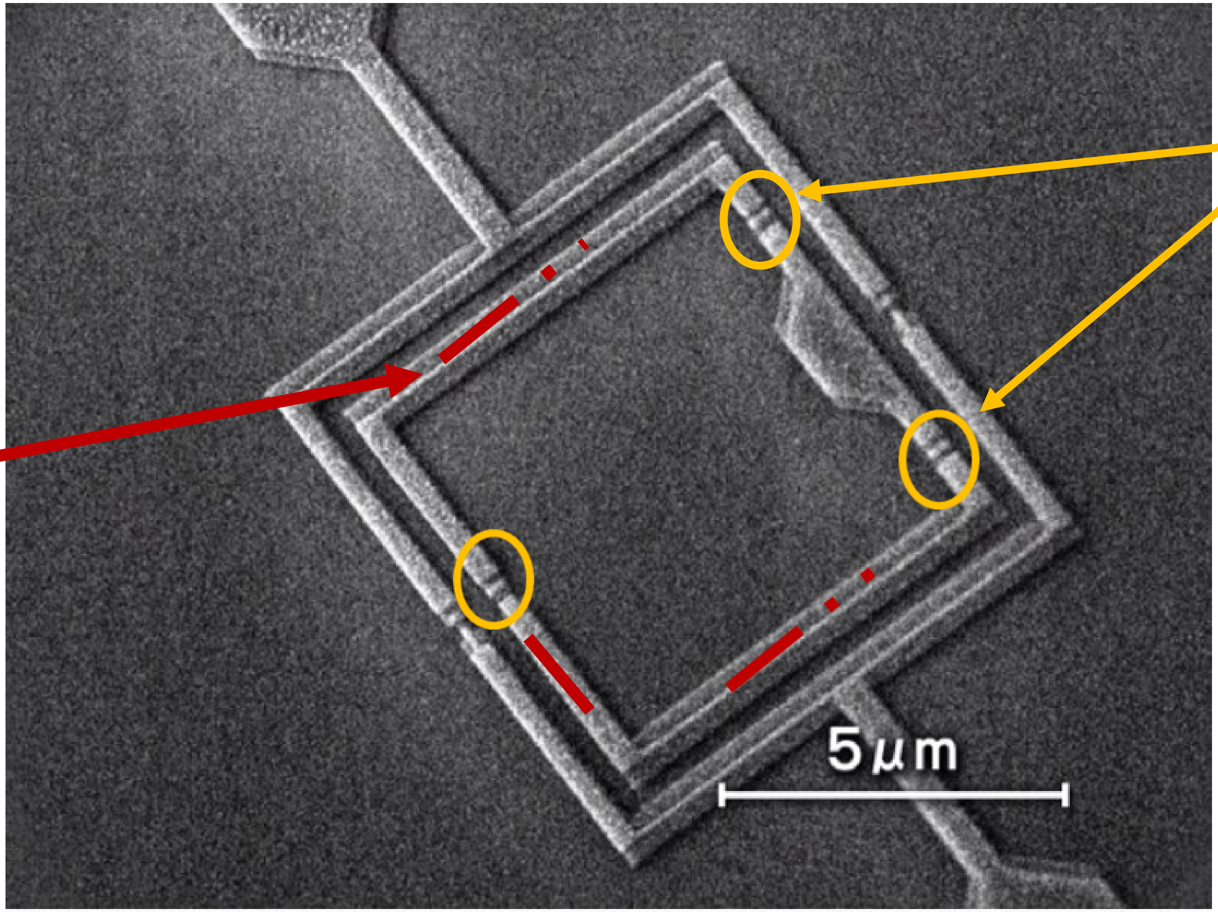
(T ~ 0.01K)

1. Microwave
2. Waveguide
3. High frequency

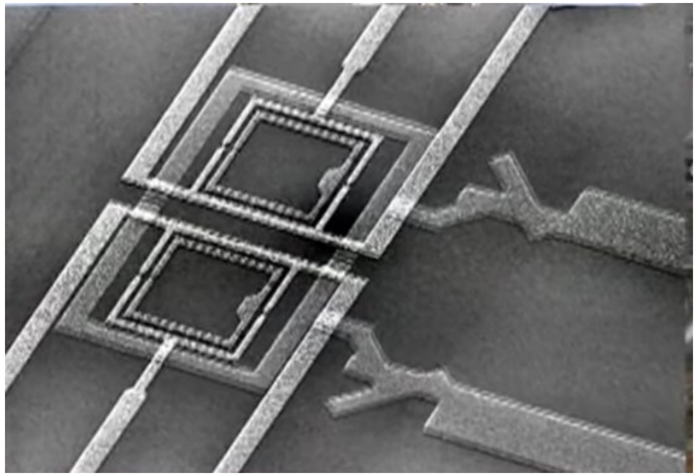
Working at Low temperature, Isolate Quantum Circuit from environments

Superconducting Qubit (Basic concepts-Example)

Alumina (AL)

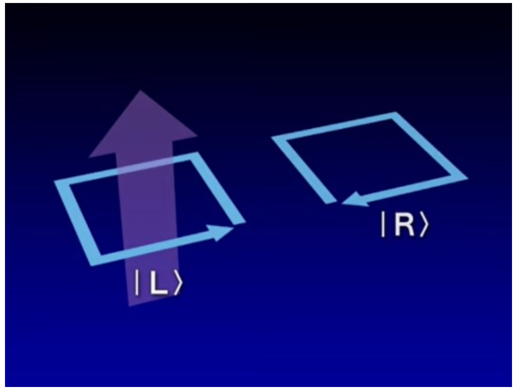


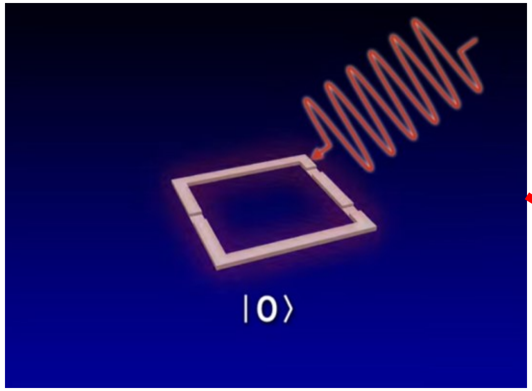
Josephson Junction(JJ):

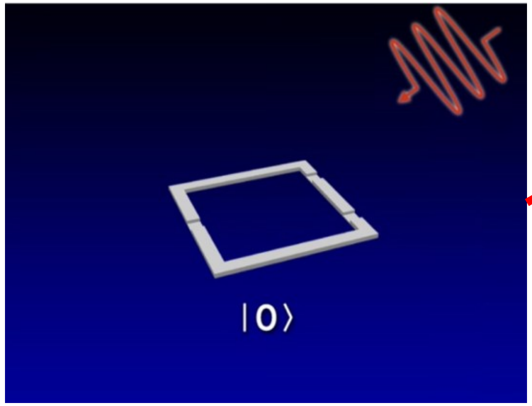


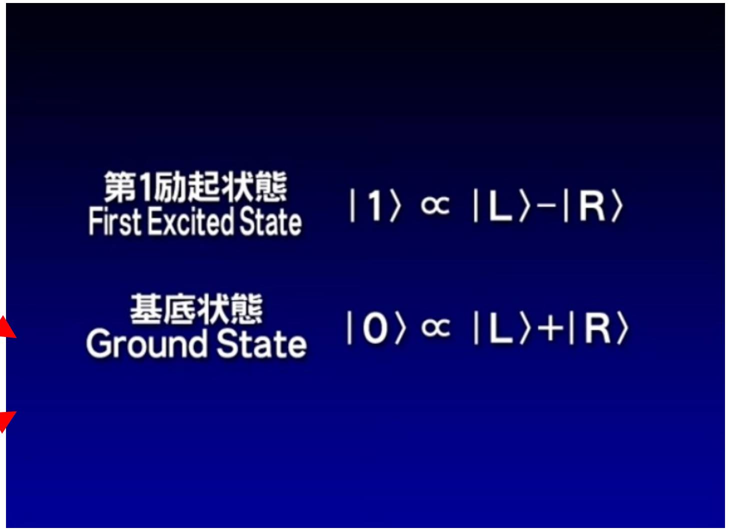
Source: NTT-Japan

Superconducting Qubit (Basic concepts-Example)









Encoding information in currents moving clockwise and counter clockwise

By changing the strength and duration of the Microwave, Superconducting Qubit creates superposition state or change states

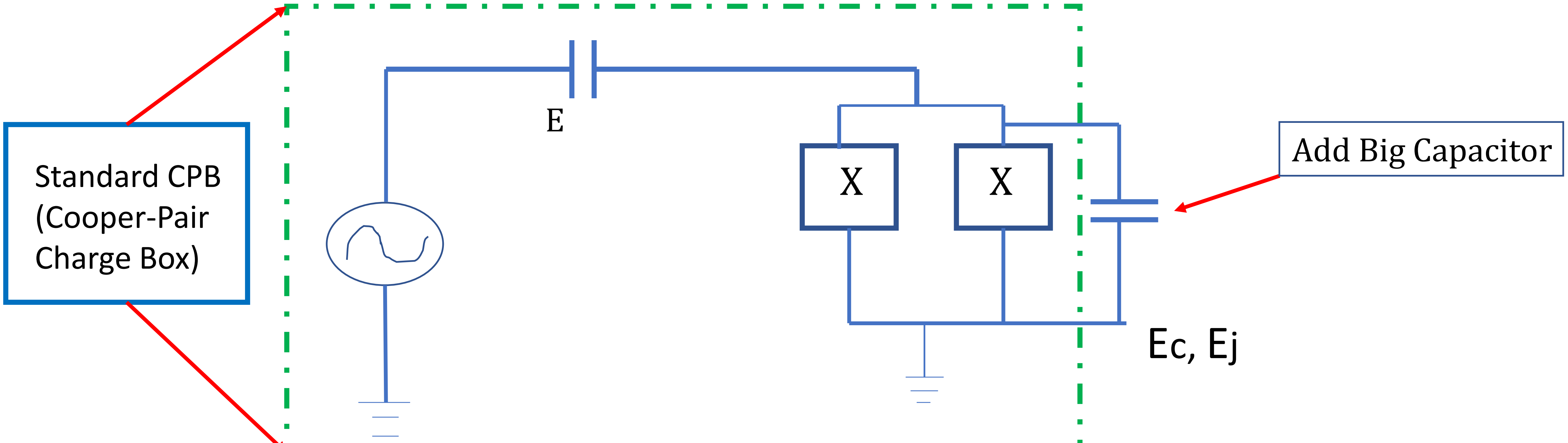
Source: NTT-Japan

Josephson Junction(JJ)-(2)

- DiVincenzo Criteria requires
  - states to initialize our system, put system in the Ground State
  - Thermal excitation don't excite the qubit out of the Ground State
- Inductance depends on the current going through the injunction
- Cold temperature
  - Typical Qubit frequency is 5 GHz/250 milli Kelvin
  - We need much colder than 250 m K, 10 milli Kelvin
- Dilution Fridges
  - Pulse tube cooler, dilution refrigerator, mixture of helium 3 & helium 4
  - Similar you cool a cup of coffee blow across the top of it, removing the vapor

Transmon Qubit – A charge Noise insensitive Qubit
(A variant of the Cooper Pair box)

- Two Huge Capacitors– Charge Qubit with two capacitors

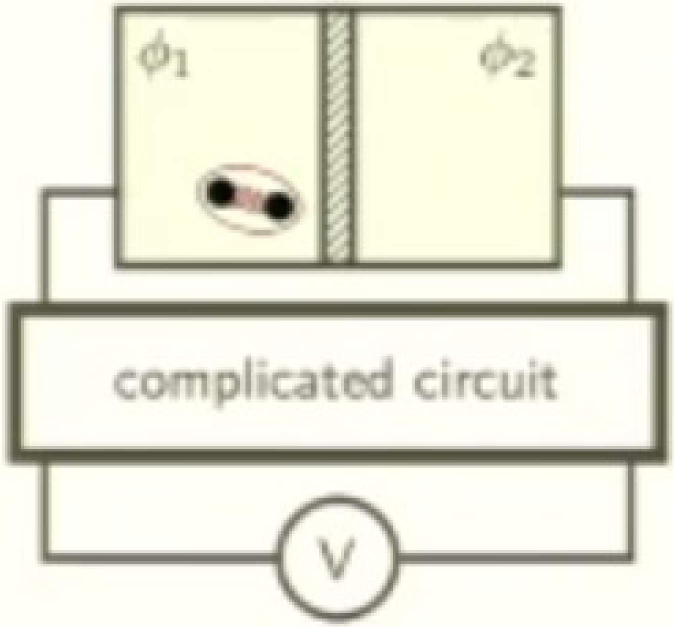


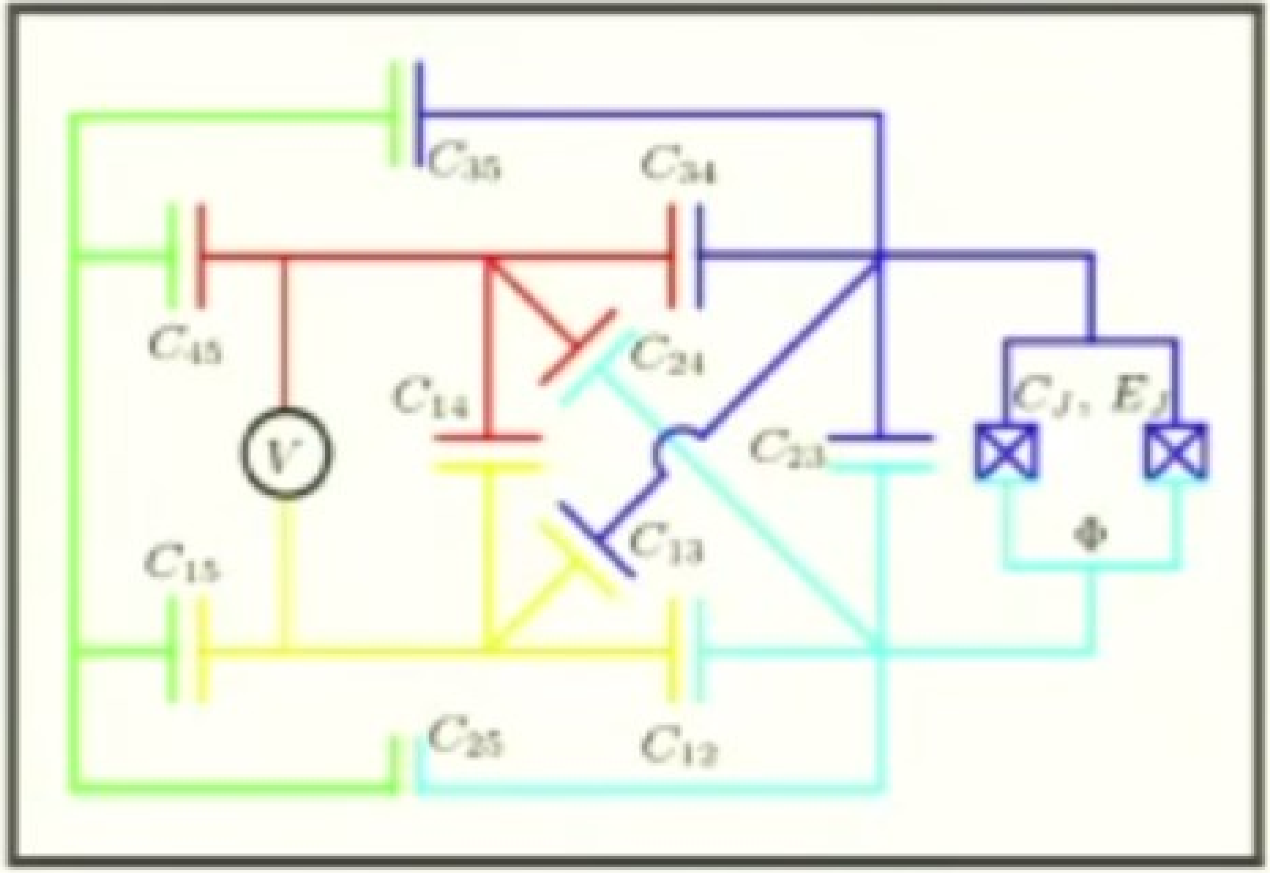
Classical circuit + small non-linear circuit
Transmon is a (good) Qubit

Transmon Qubit
(Transmission-line Shunted Plasma Oscillation Qubit)

- Jens Koch, Andrew Houck
- CPB shunted by a large capacitor
- Frequency tunable Transmon qubit
- Transmon is a noise insensitive qubit
- Coherence time $\sim$ hundred micro second, ($\sim$ 100 us)

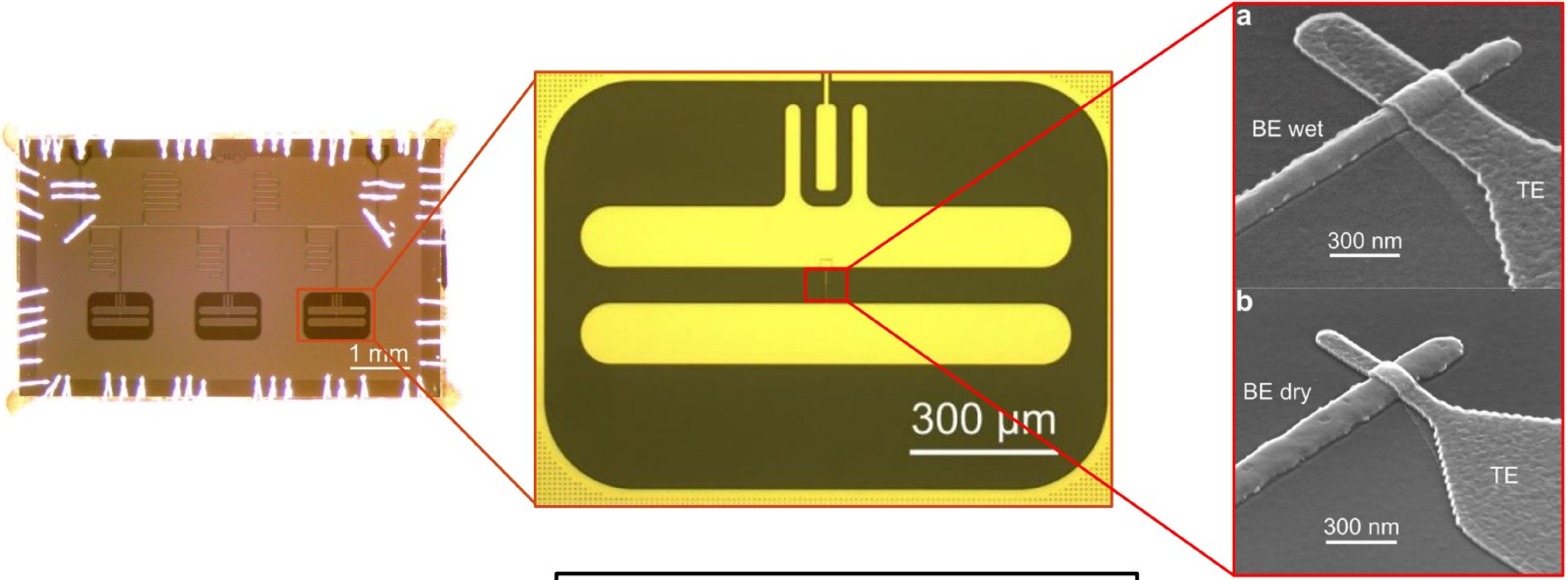
$$\hat{H}_{ST} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\phi}$$





J. Koch et al., Phys. Rev. A (2007)

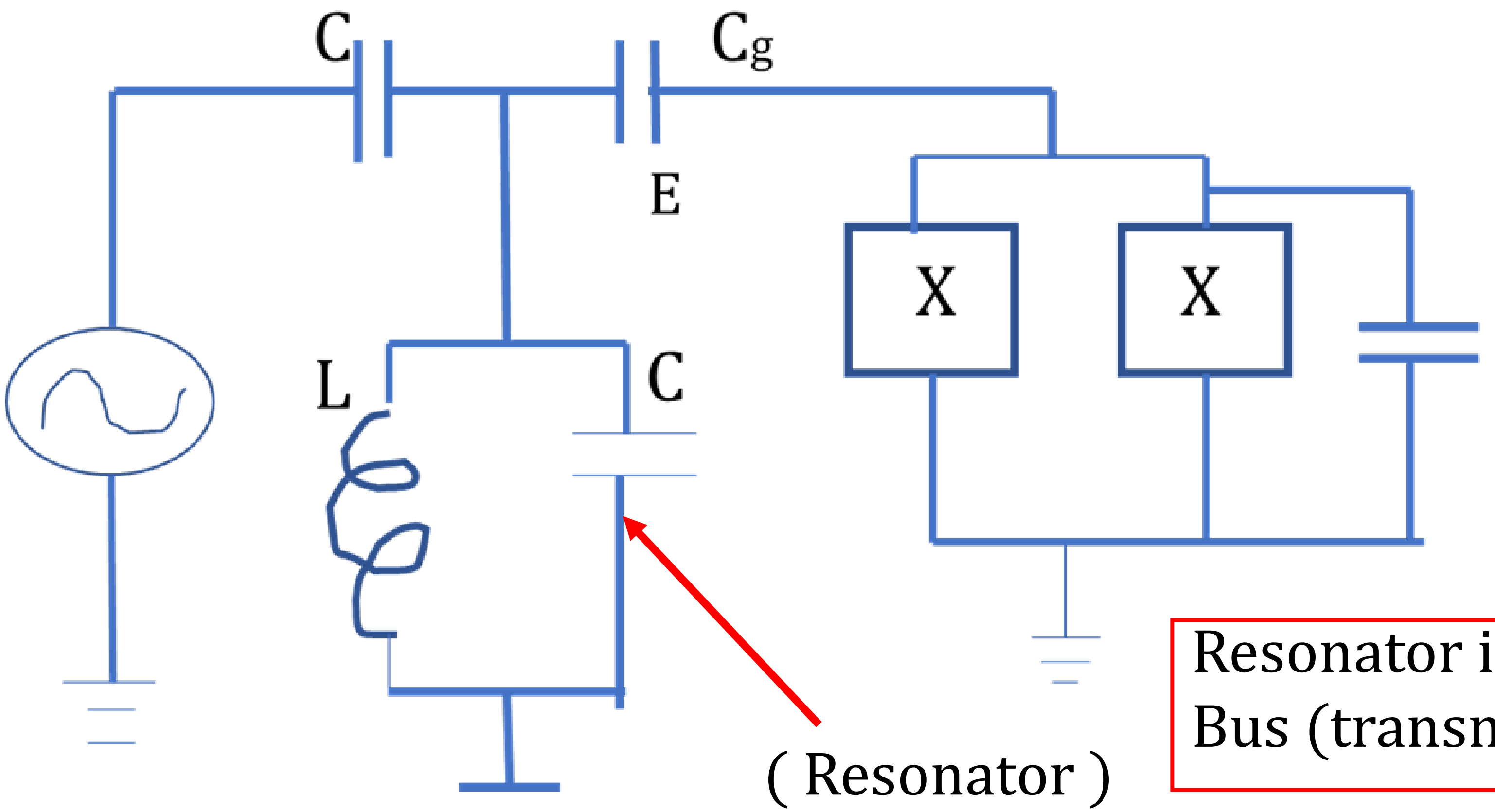
High-coherence transmon qubits



T_1 times exceeding 0.1 ms

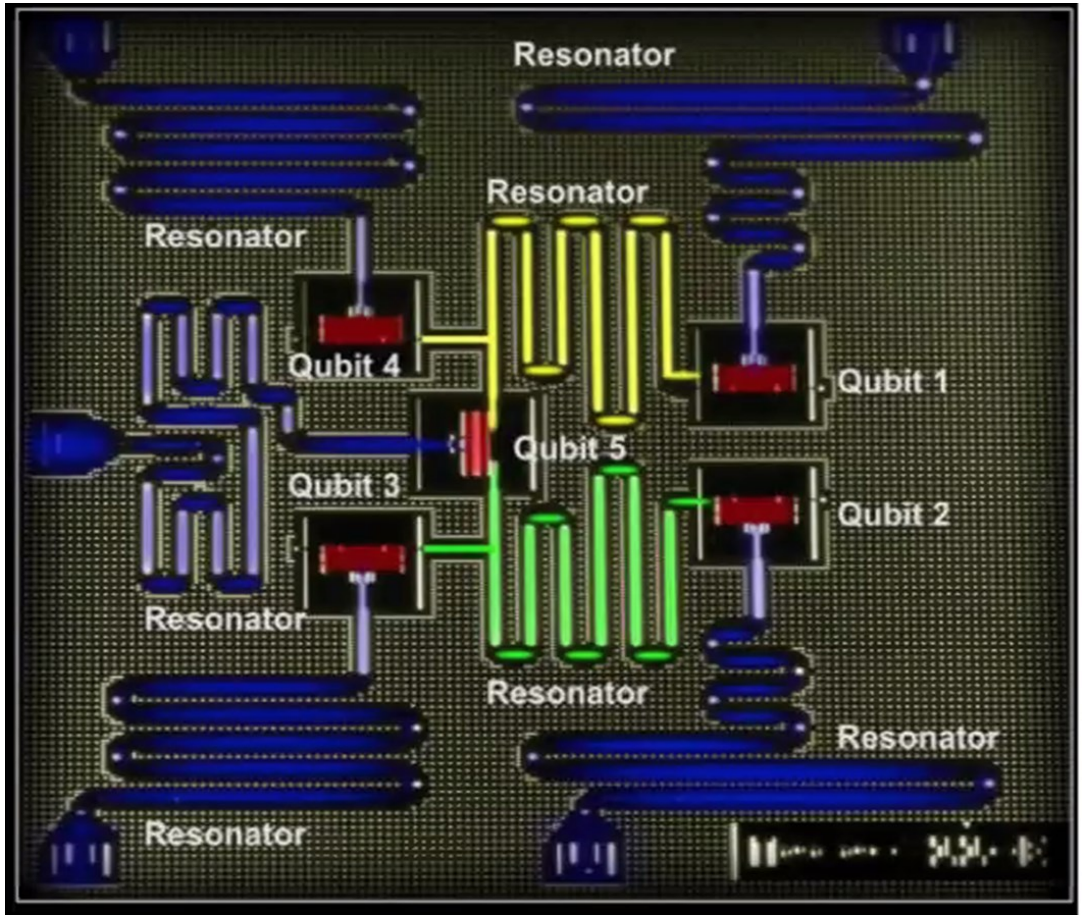
Overlap Josephson junctions 300mm fab compatible: Verjauw, Jeroen, *et al*. https://doi.org/10.48550/arXiv.2202.10303

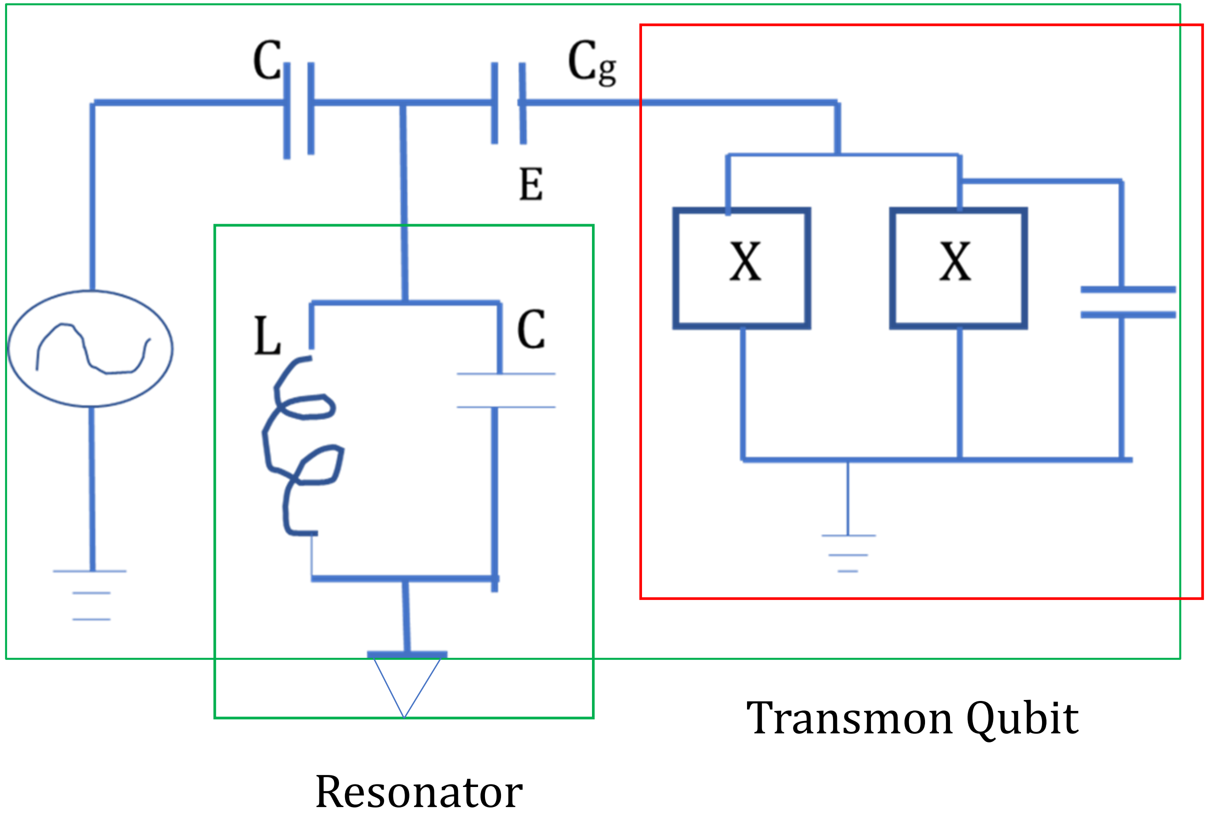
Qubit coupled to Resonator



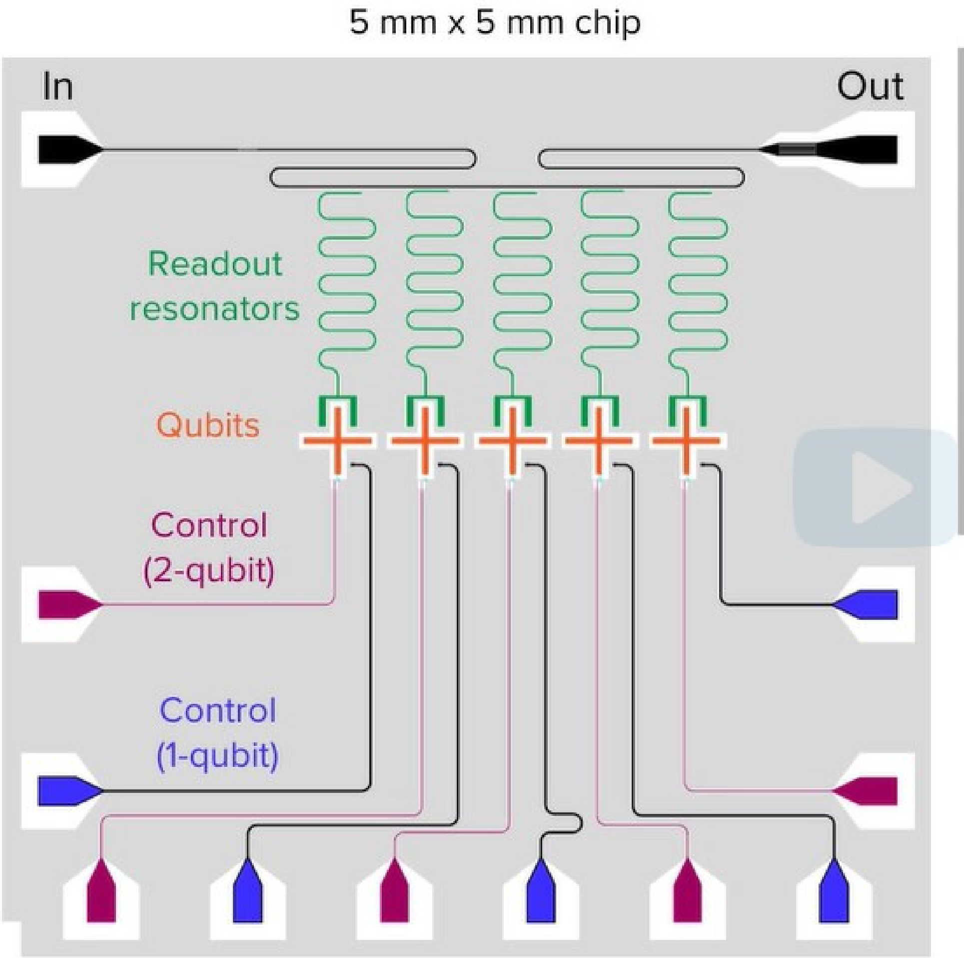
Resonator is the Quantum Bus (transmission lines)

Transmon Qubits Physical Layout and Circuit Model

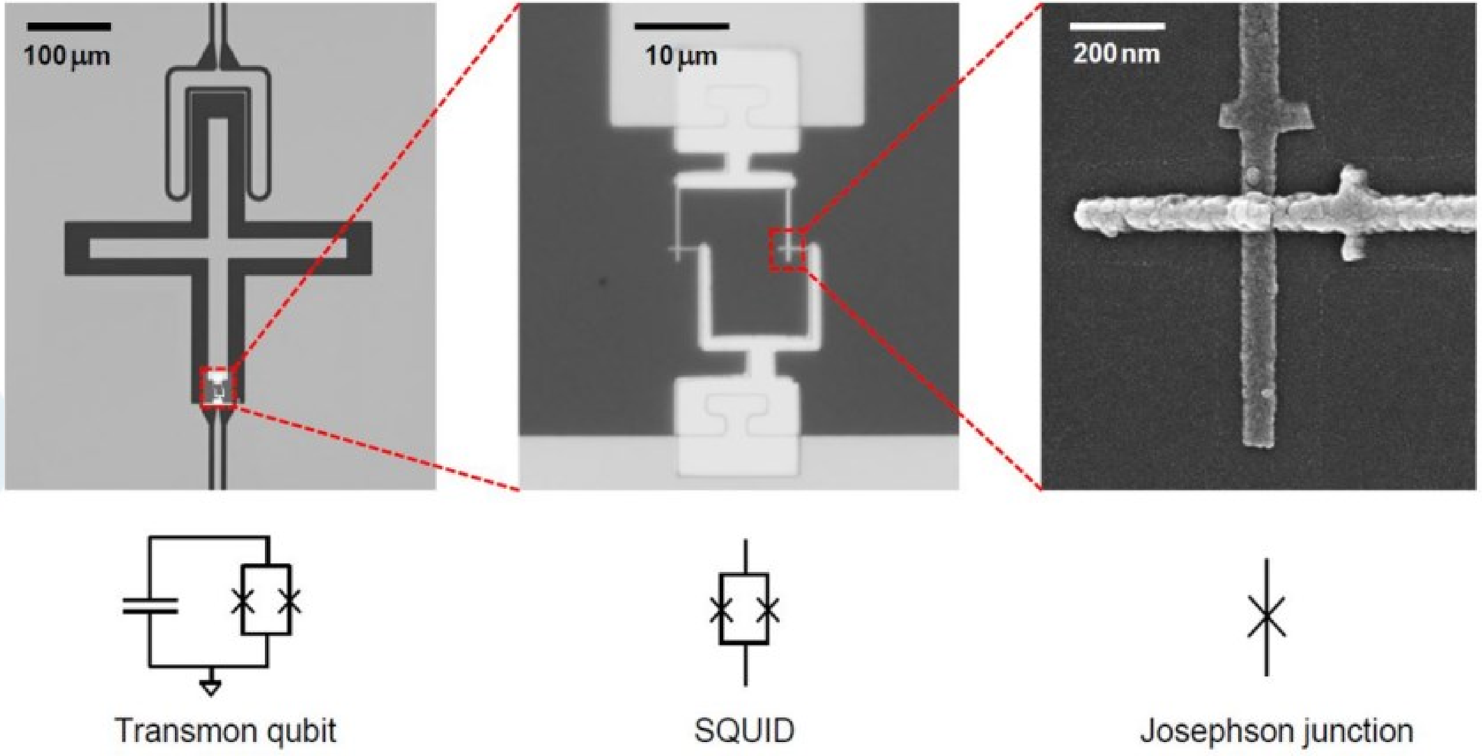




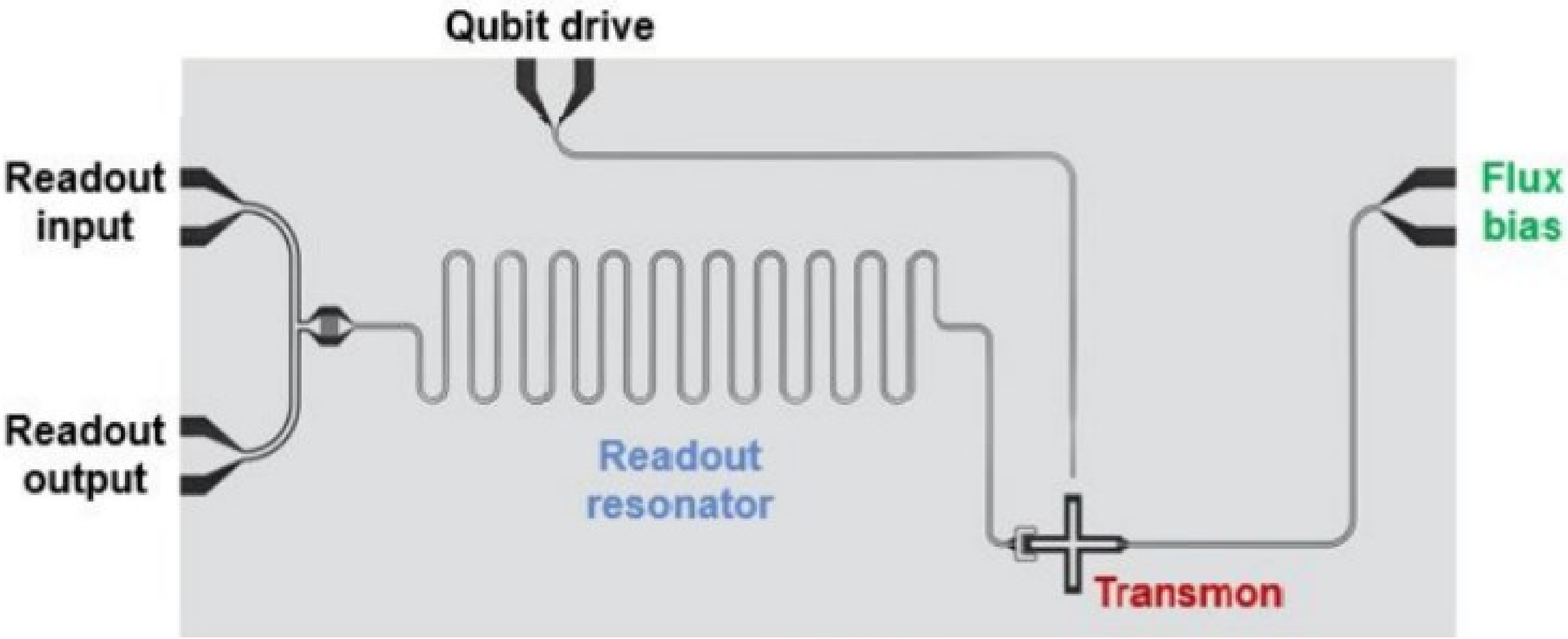
Source: IBM

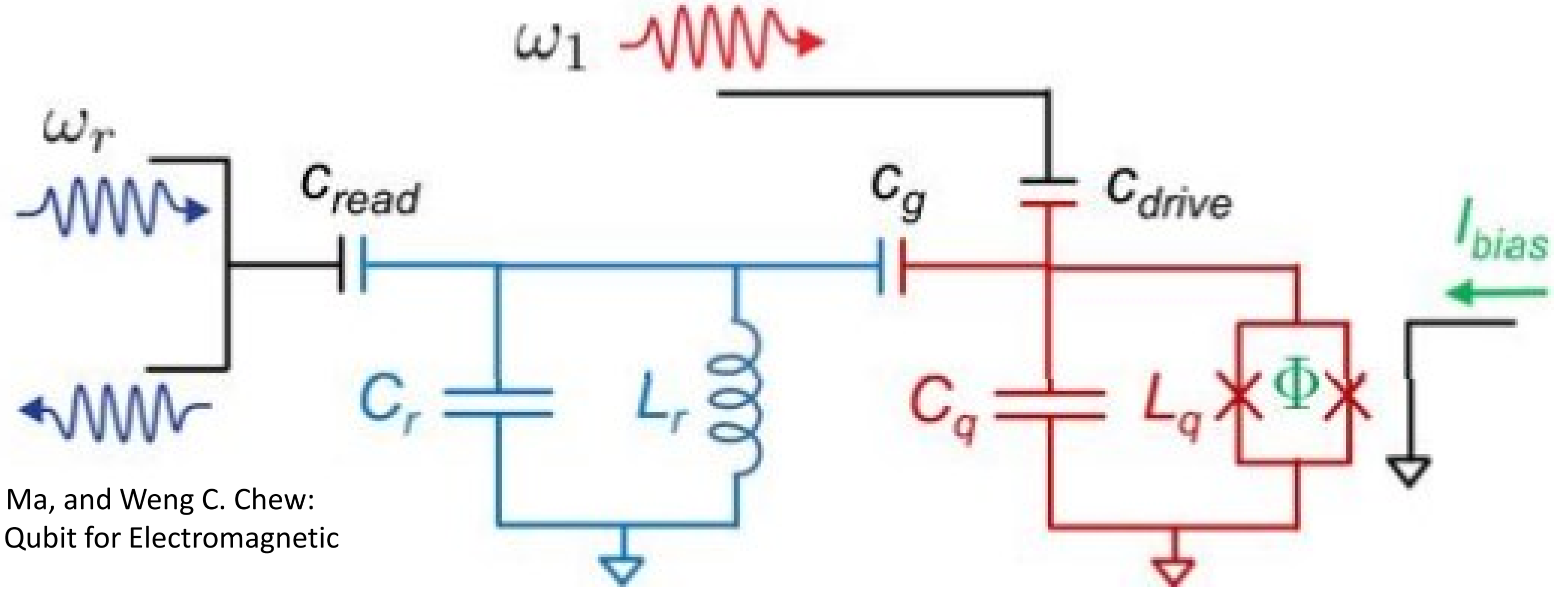


Source: MIT



Source: Thomas E. Roth, Ruichao Ma, and Weng C. Chew: An Introduction to the Transmon Qubit for Electromagnetic Engineers(See Ref. 11)

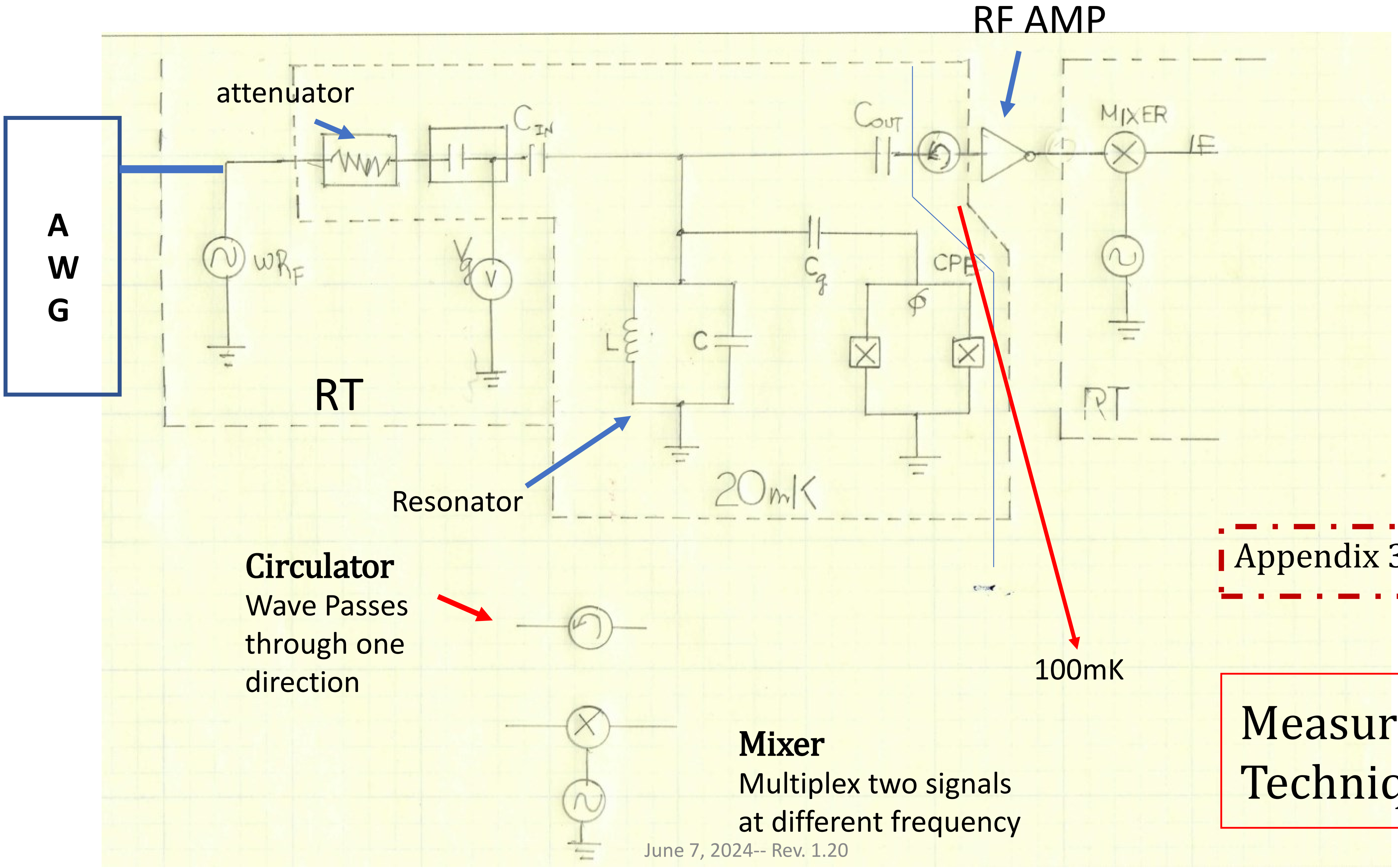




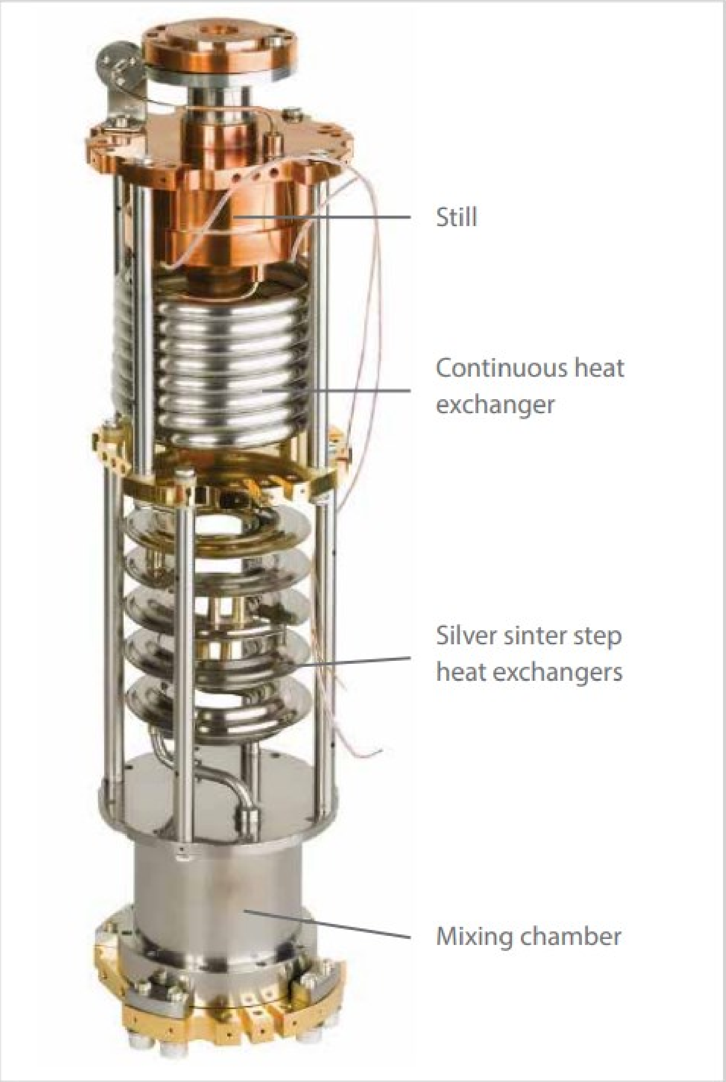
Source: Thomas E. Roth, Ruichao Ma, and Weng C. Chew: An Introduction to the Transmon Qubit for Electromagnetic Engineers

Measurement Technique

Appendix 3



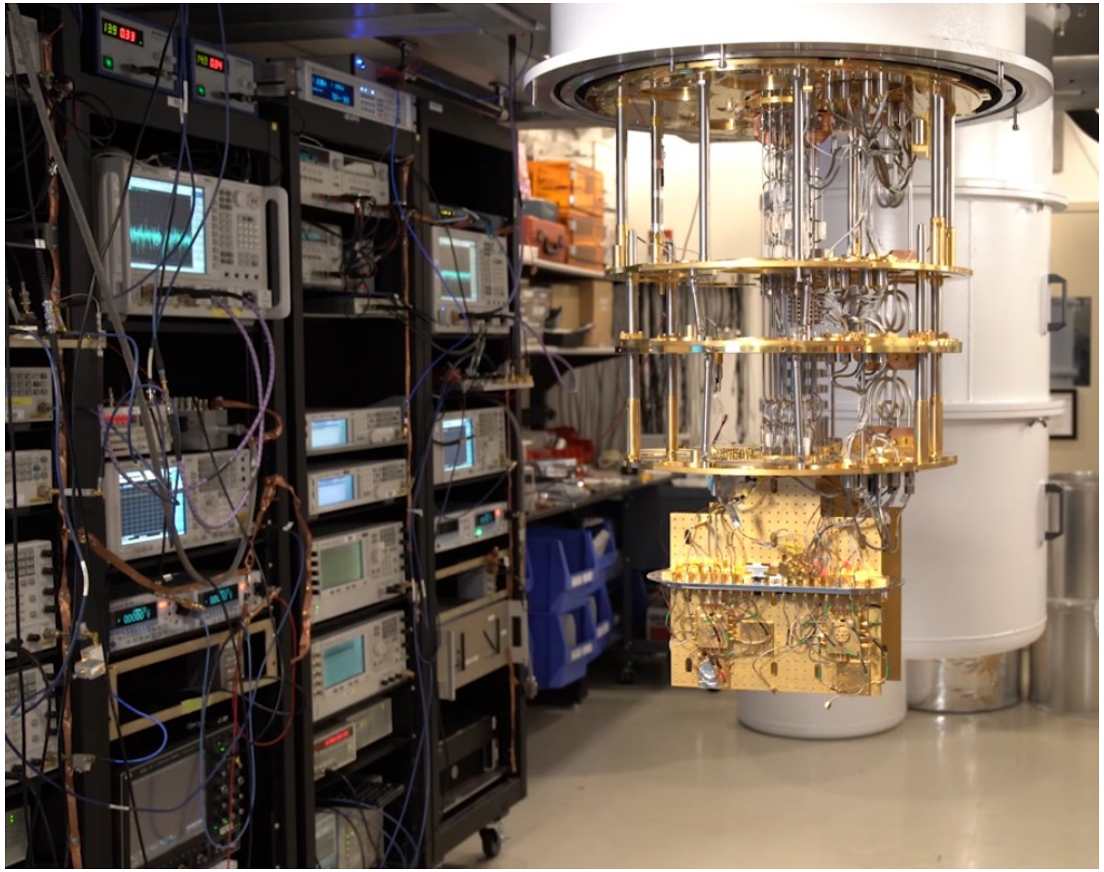
(Dilution Unit- below 10 m K)



Dilution unit for operations below 10 mK

Source: Oxford Instruments

Dilution Refrigerator

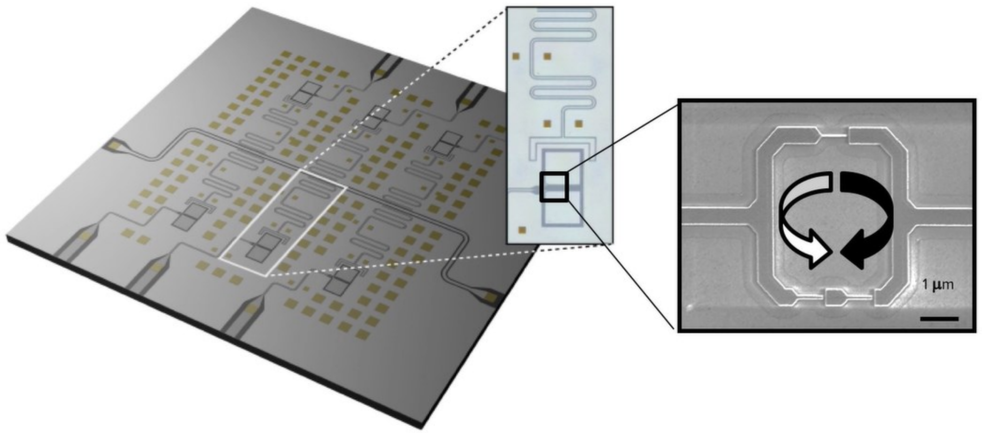


Source: MIT Lincoln Lab.

Superconducting Qubits-Control

- To control Qubit—Sending Pulse of Microwave energy at Qubit's frequency
- Control pulse– Wavelength and phase

Controlling Superconducting Flux Qubits



Source: MIT Lincoln Lab.

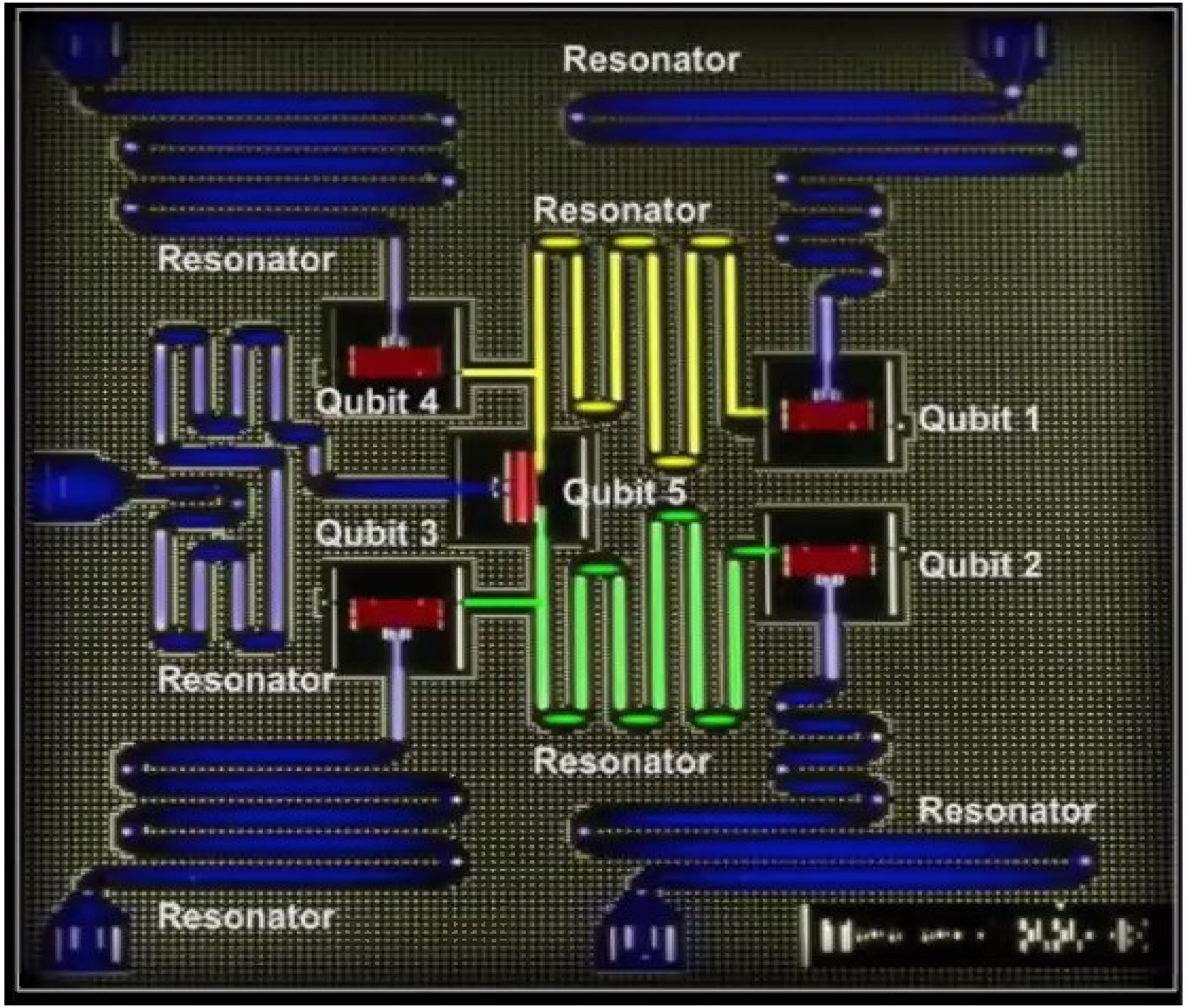
Superconducting Qubits-Control (2)

- Control Instrument is an Arbitrary Waveform Generator
  - Single qubit control pulse is about ten’s of nanoseconds, very short compare with Coherence times of 100us.
  - Fidelities $> 99.9\%$
- Readout Resonator —Information out of Qubits
  - Microwave transmission line has distribute inductance and capacitance is also a Resonator.
  - Sending a pulse of Microwave energy to interrogate the resonator, we can leaner the state of the Qubits, Readout time is 100ns.

Superconducting Qubits-Testing Steps

- Testing--- Wafter Sort Testing (Defect Free)
- Qubit Loop: E-beam system <10nm, Stepper Lithography
- Room Temperature Testing – Cryogenic Testing
  - Josephson Junction (JJ)
  - Metal layers– Measuring current density of Josephson junction and measuring contact resistance (contact chains)

IBM
5-Qubit
Quantum
Processor



Quantum Computer –

Qubit testing, assembly, Qubit characterization, and Cable's electrical characterization

1. How to characterize qubits(Superconductor qubits and other qubit modalities) ?
2. How do you assemble all the cables/wires?

   a. Robot? b. Manual? c. PCBs

3. How do we characterize the cable's electrical characterization?
4. CMOS Technology –IC Design, Cryogenic CMOS5.
5. How to test the resonator?

The subjects are research of interest
in Engineering and Production.

Superconducting Qubits --- Review

- Quantum Engineering on the Qubit of Quantum Computer
  - Quantum Control Engineering
  - Engineering a high fidelities Qubits (Flux, Phase, or Transmon)
- Quantum Engineering on the Qubit's Read-out fields.
  - Cryo-CMOS
  - Single Flux Quantum Logic
  - Parametric Amplifiers
- Gate Time: for a single operation
- Coherence Time: The lifetime, Environmental disruptions
- Threshold $\sim 10^3$ (Figure of merit)
- Superconductor : 10ns = Gate time, 100us= Coherence time
- Trapped Ion: 10-100ns = Gate time, 1~50s = Coherence time

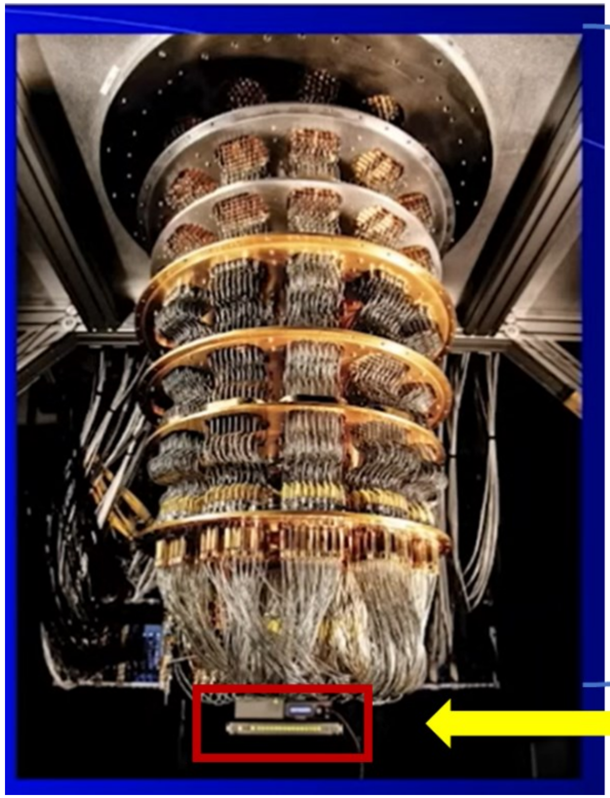
Quantum Computer (Superconductor Qubits)
Hardware Design Guidelines

(Learning from Classical Computer Design)

(Engineering View)

Fundamental Structures of Quantum Computer Hardware Design

- Quantum Computer Hardware structures have three function blocks
  - a. Quantum Processor Unit (QPU) consists of Qubits silicon and other elements.
  - b. Communication Links: Qubits Controls, Readout, Measurements etc. The links operated under low temperature to room temperature, and
  - c. External (room temperature) control units and computers, etc. (Quantum State Controller)



Communication Links:

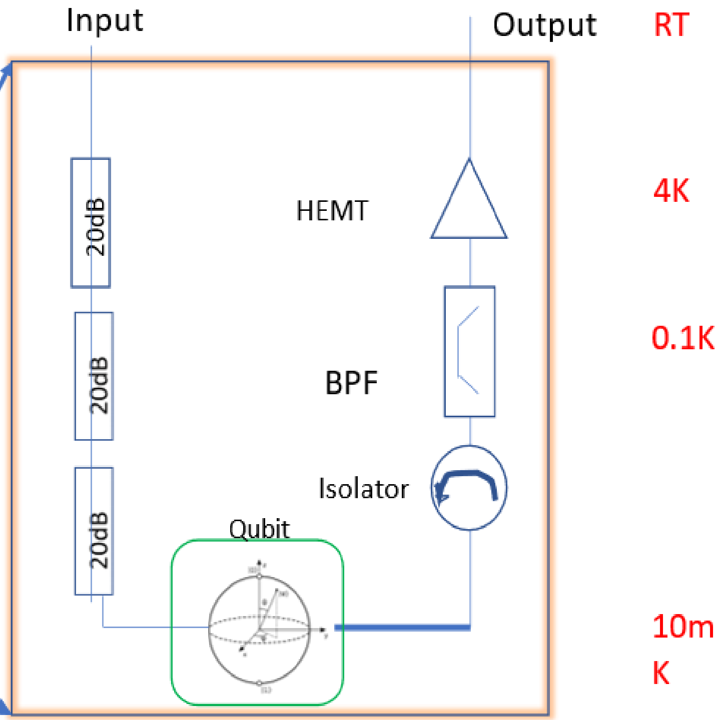
Qubits: Controls, Readout, Measurements

Quantum Processor Unit (QPU) Superconducting Qubits

Superconducting Qubit

Testing

Dilution refrigerator



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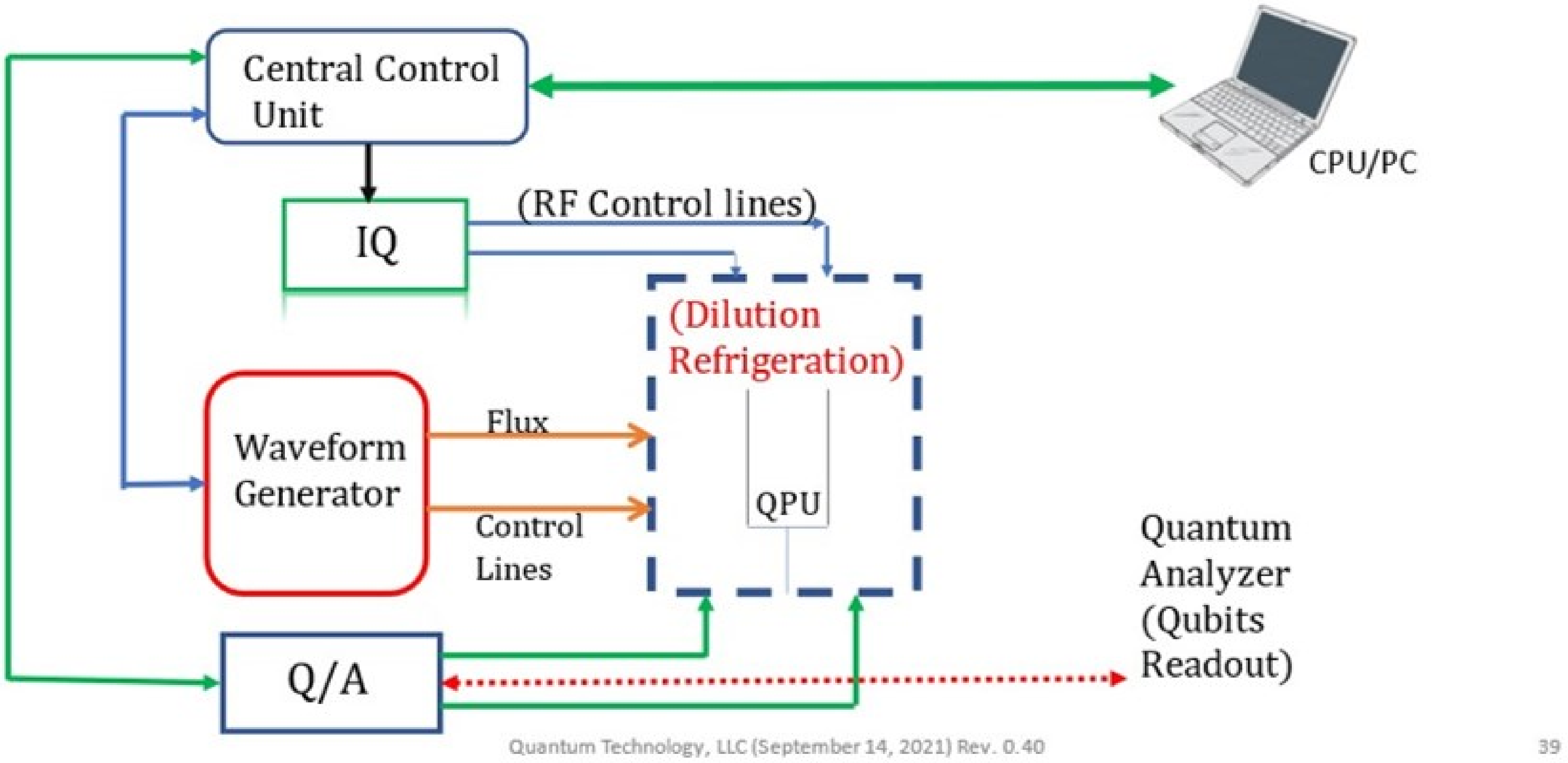
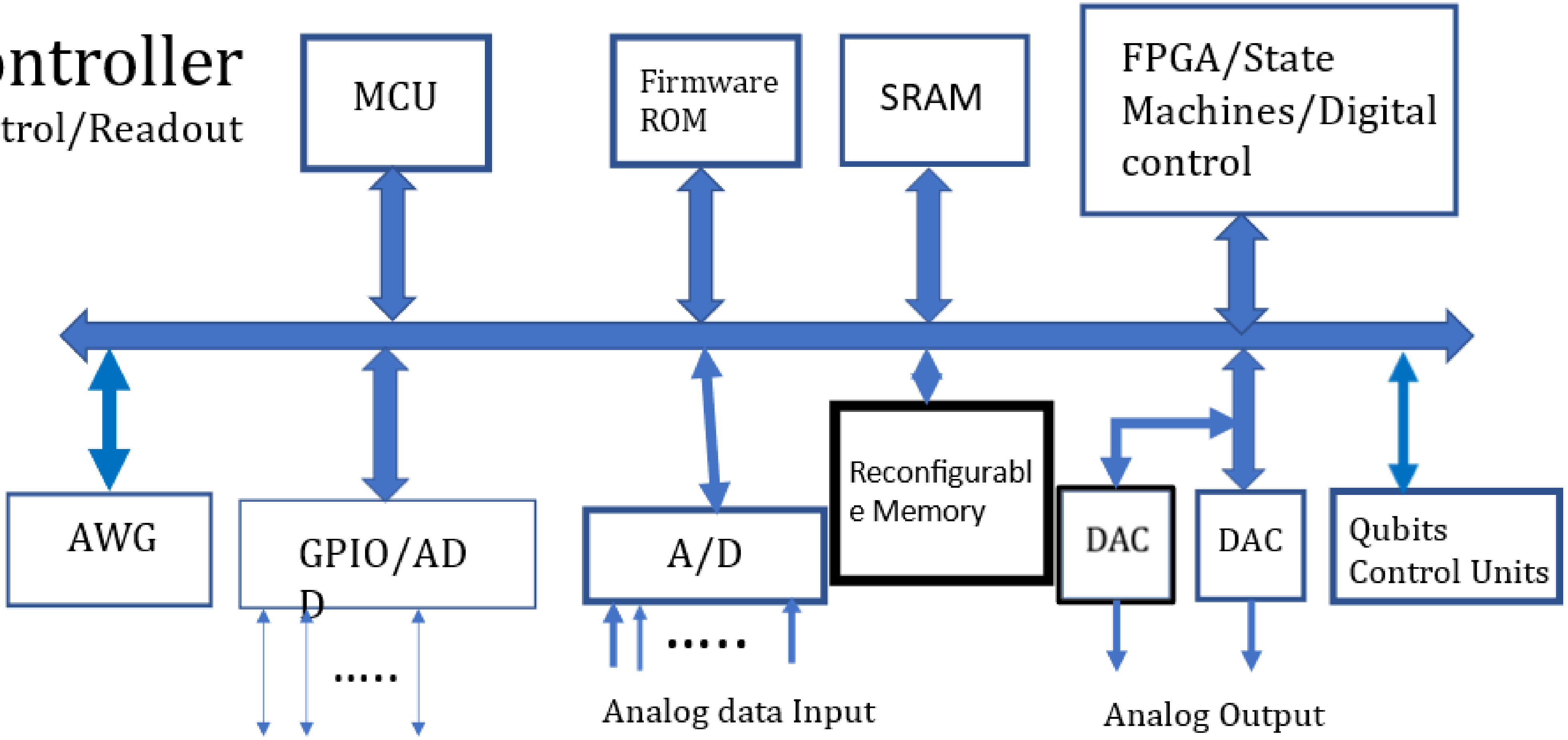
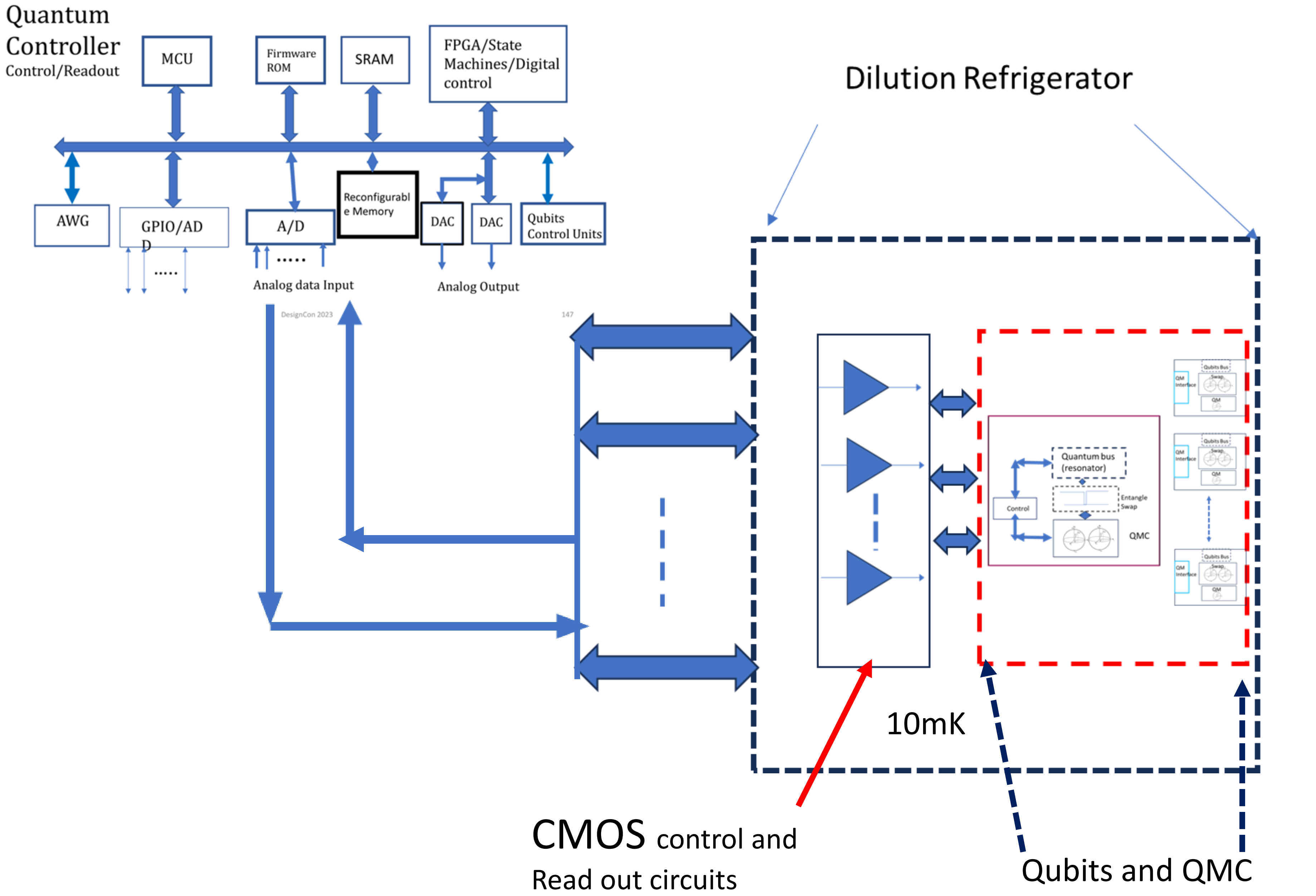
Quantum Computer Controller Block Diagram



Quantum Controller
Control/Readout

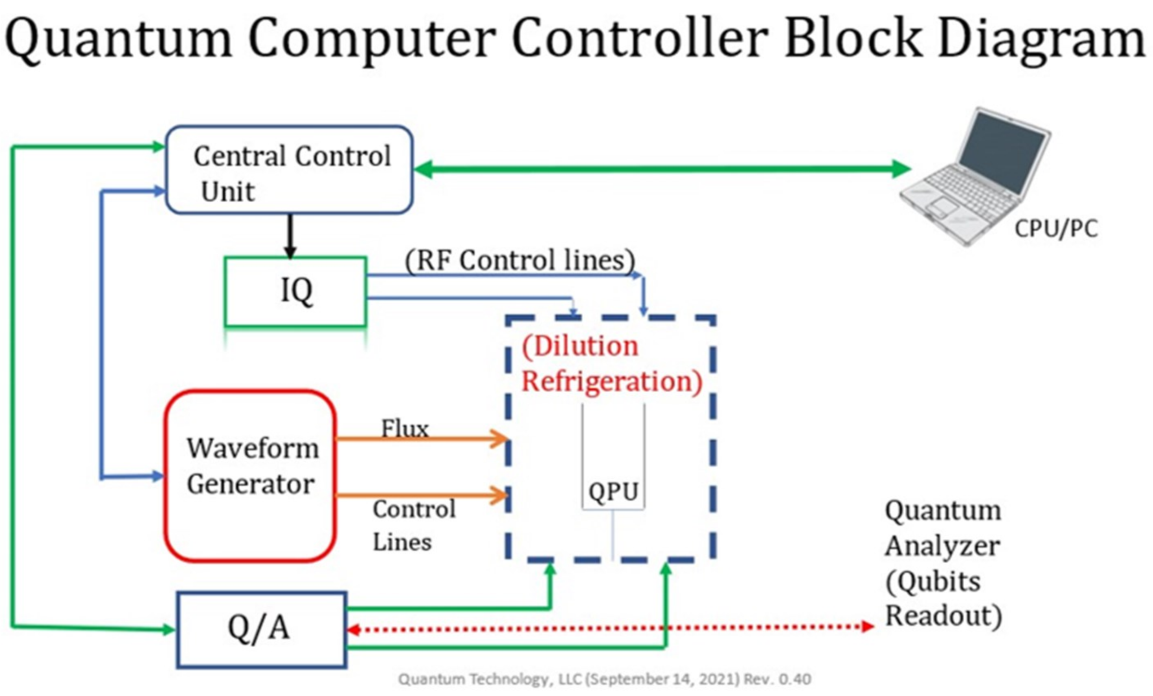




CMOS control and Readout circuits

Inside of Outside?

Example of Quantum Computer and Functional diagram



DesignCon 2023

Room temperature: External Quantum computer Controller

Source: IQM 20 qubit QC

Scale Issues—Space and Electrical problems

What are the potential problems of the circuit/cable connection?

Long wires/cables---- cannot scale properly.
Requiring New Design (Packages)

Example: 1-qubit needs 5 wires to access one qubit
5-qubit needs 25 wires

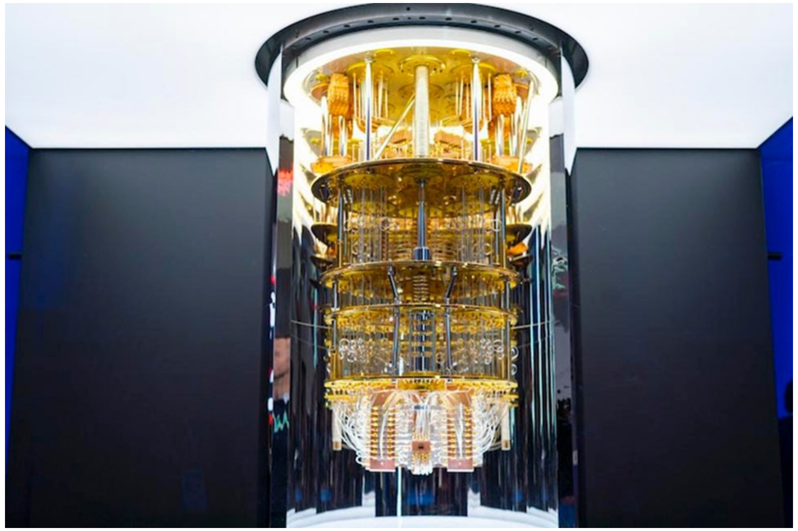
1000-qubit needs 5,000 wires

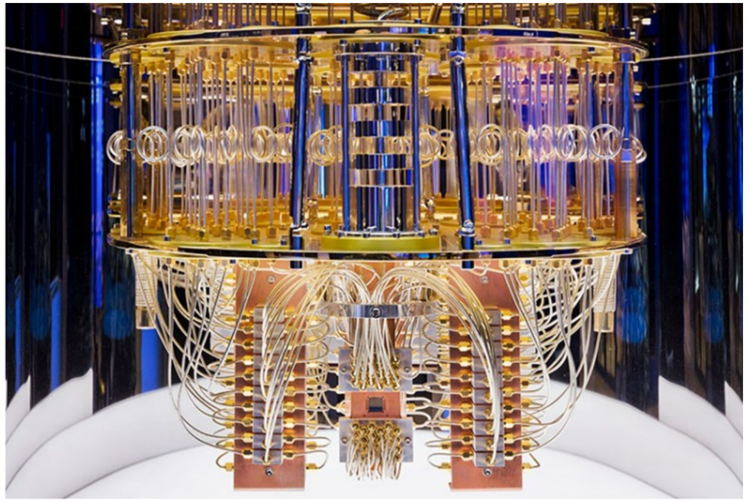
1,000,000-qubits needs 5,000,000 wires/cable
[Note: space/electrical problems.]

Electrical problem: noise, signal quality degradation, accuracy issue.

It is untannable for QPU with a large number of Qubits.

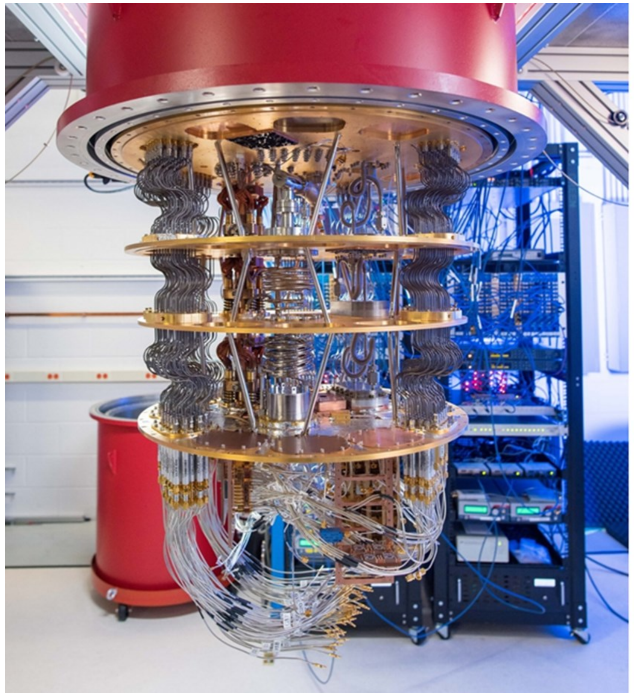
- A proposal (recommendation) to replace the entangled cables for Quantum computer hardware design and improve the manufacturing yield.





The innards of an IBM quantum computer show the tangle of cables used to control and read out its qubits. Credit: IBM

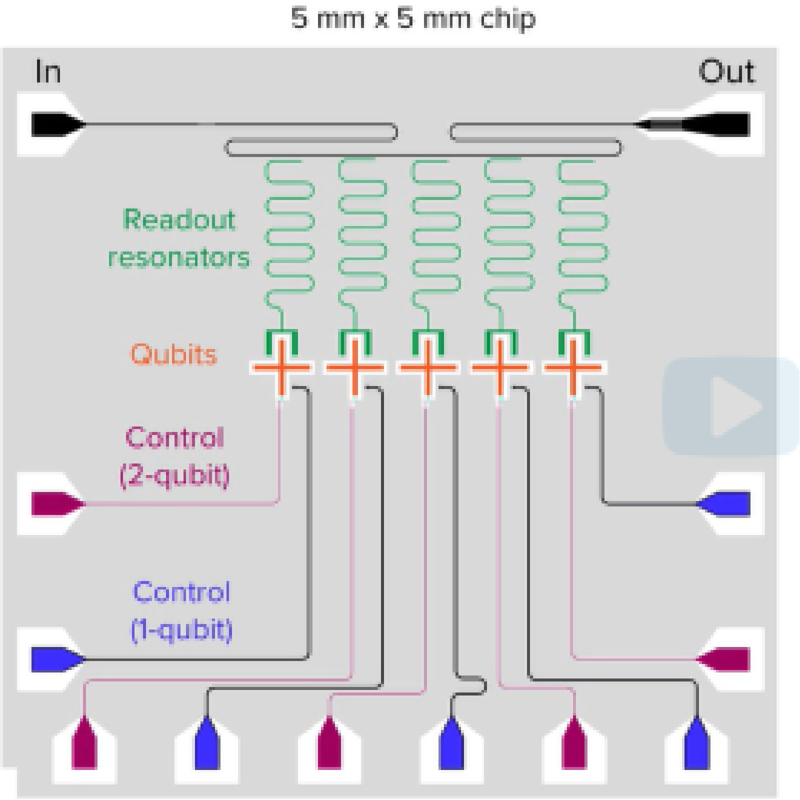
QPU and Cables within Dilution refrigerator, IBM Quantum Computer

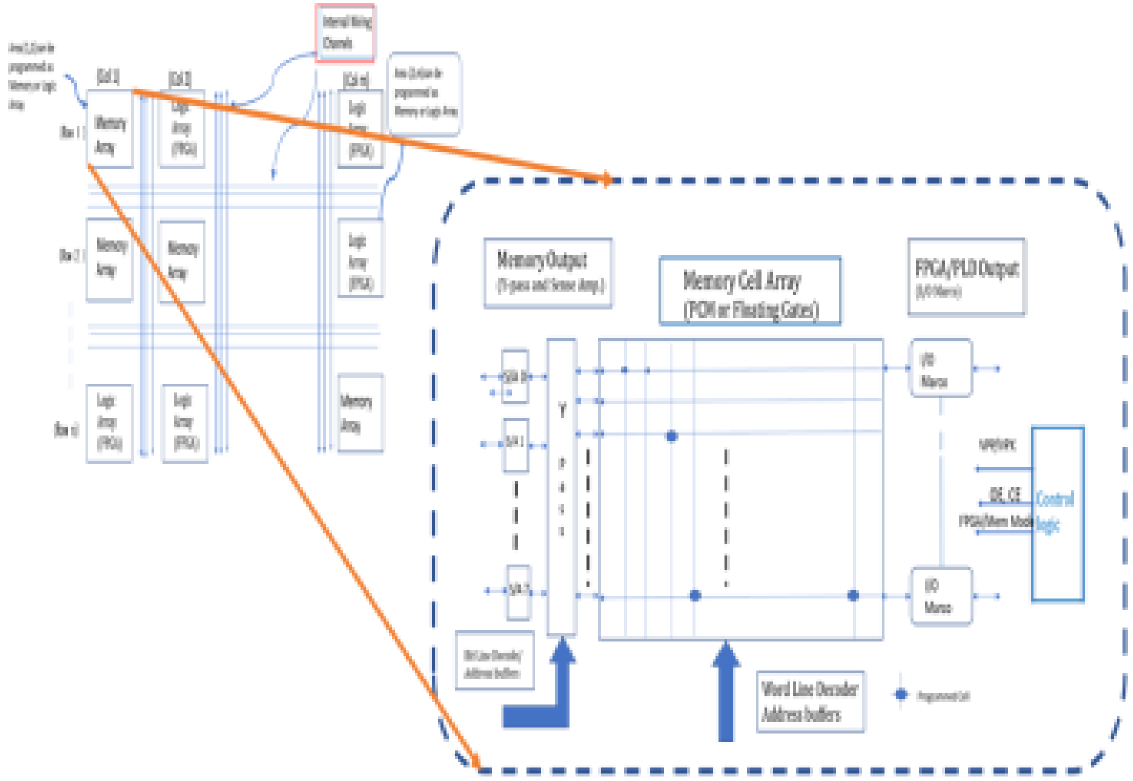


QPU and cables
Source:
https://de.wikipedia.org/wiki/Quantencomputer

Qubit Arrays vs Classical Memory Arrays

- Qubit Arrays
  - Requires RF pulses in the 4 to 6 GHz to manipulate their states
  - Control line for every Qubit at room temperature (RT) to 10mK(-273.14 ^{0}C)
  - Control and Read-Out circuits have to access each Qubits from RT to low temperatures.
  - Each Qubit is coupled by Resonator.
- Classical Memory Arrays
  - Memory cells form as a Matrix, peripheral circuits, X-Decoder, Y-Decoder to access the selected cells. No induvial control lines are required.





Qubit cell array

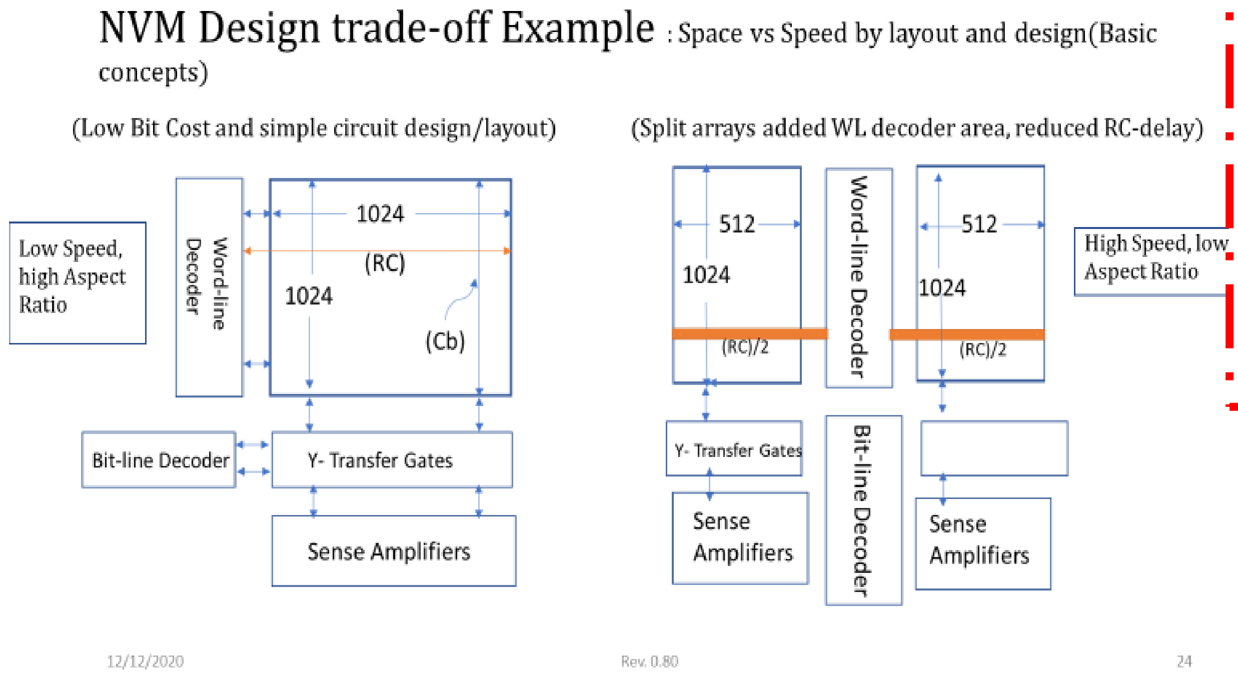
Classical Memory cell array

Source: MIT

Source: W. Liu

Qubit Array vs Classical Memory Array

5 lines to access one Qubit



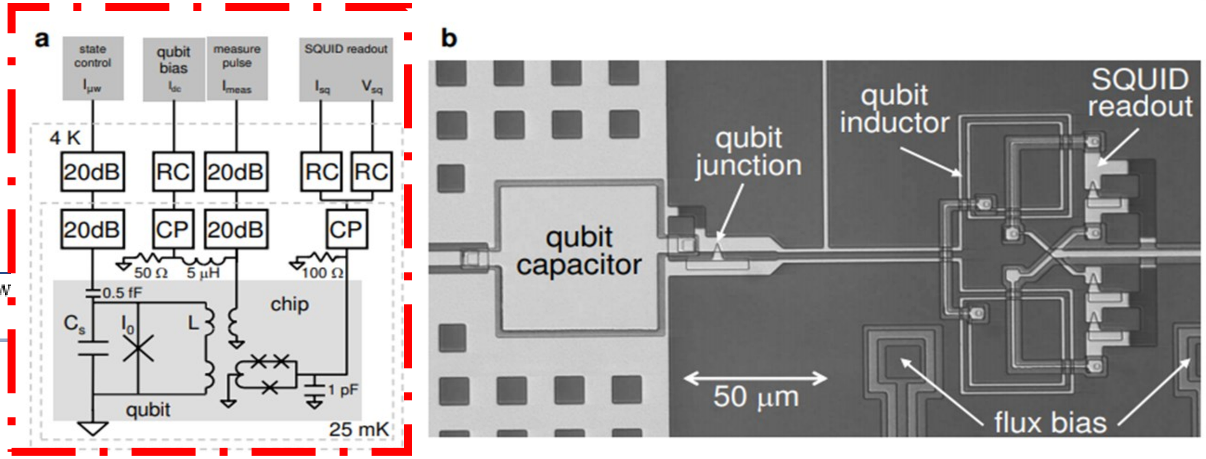


Fig. 2 (a) Schematic diagram of a phase qubit circuit and bias lines. Symbols 20 dB, RC, and CP represent 20 dB microwave attenuators, resistor-capacitor low-pass filters, and copper-powder microwave filters, respectively. The RC filters for the qubit and SQUID bias has 1 kΩ and 10 kΩ DC resistance, respectively, and a roll-off frequency $\sim$ 5 MHz. The 5 μH inductor is a custom made radio-frequency bias tee with no transmitting resonances below 1 GHz. (b) Photomicrograph of present phase qubit, showing small area ($\sim 1\,\mu m^2$) junction shunted by a parallel plate capacitor. Microwave drive line (with capacitor, not shown) comes from the left. The qubit inductor is coupled to a SQUID readout circuit in a gradiometer geometry. The flux bias lines for the qubit are symmetrically placed about the SQUID and counter-wound to inhibit flux coupling to the SQUID. The SQUID bias line exits to the right. The holes in the ground plane inhibit trapped vortices in the superconducting ground plane.

Quantum Computer– Superconductor
Qubits requirements

- High Gate fidelity, long coherence time, and Fast gate speed
- Large number of Qubits array– Space, scaling issues
- Analogy and Digital control/read-out circuits, microwave control circuit
- Low temperature– 10 mK (-273.14 ^{0}C)
- CMOS transistor model at low temperature
- Wire's electrical characteristics
- Superconducting Qubits
    - Josephson Junction (JJ) and Resonators (Quantum Capacitors, and Inductors)

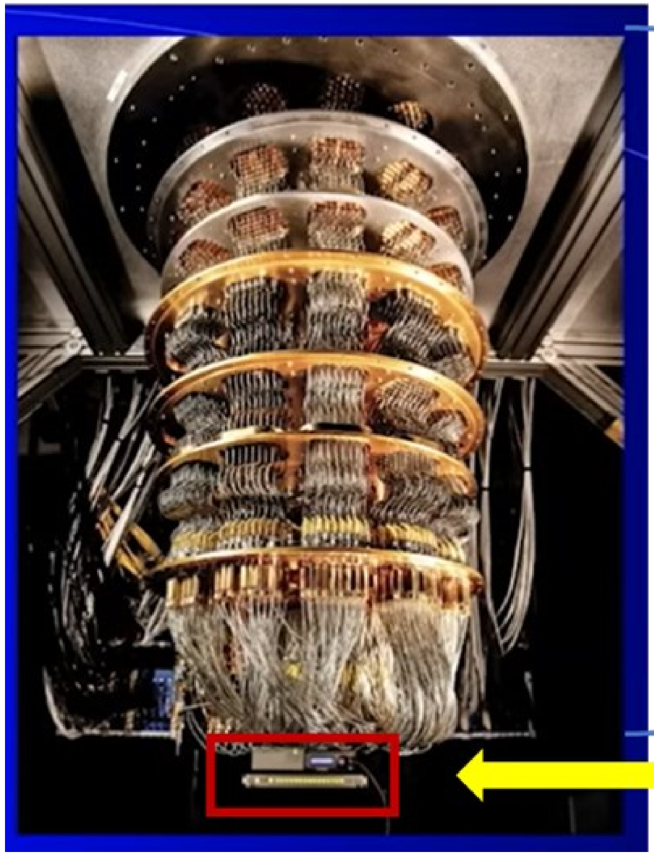
- 72 Qubit quantum processor requires:
  - 240 high speed AWGs
  - 84 upconverters
  - 12 downconverters
  - 24 high speed ADCs
  - 168 coax, temperature: 300K to 4K
  - 168 superconducting coax (4K to 10mK)
  - >3Tb/s data stream

Source: Google AI Quantum

Joseph C. Bardin et al., “Design and Characterization of a 28-nm Bulk-CMOS Cryogenic Quantum Controller Dissipation Less Than 2 mW at 3K”

Quantum Computer Hardware Design's Challenges:

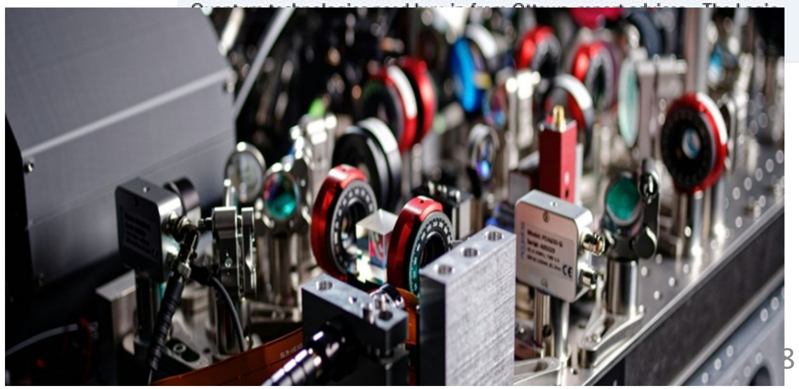
- How to scale up Quantum Computer?
  - Large numbers of interconnect/entangled cables of electronic circuits to control and measure Qubits create a bottleneck for Quantum Computer to scale to Large Quantum Computers.
  - Qubits must operate at low temperature, T= 10mK.
  - Cryo-CMOS
  - Control and measurement circuits operating under low temperatures.
  - Low power



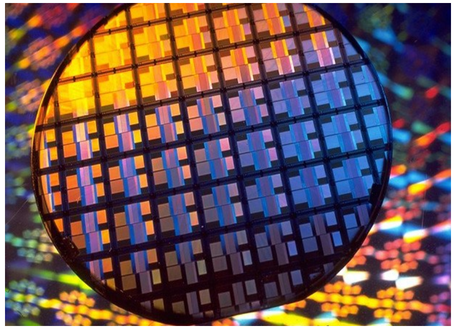
Communication Links:

Qubits: Controls, Readout, Measurements

Quantum Processor Unit (QPU) Superconducting Qubits



?



- ❑ Building Quantum Computer hardware has many challenges;

  - ✓ Low Temperature (milli Kelvin Temperature)
  - ✓ Integration of control and readout that maintain Qubit coherence at low temperature
  - ✓ 3D integration technology is required, and

- ❑ Strong Semiconductor Technology knowledge

Summing Up:

- Today's Quantum Computer company has three parts of expertise: building semiconductor chips including software, manufacturing the QC hardware (assembling into compact package), and Quantum physics.
- The semiconductor (chip) company won the market from the mini-computer and later the supercomputer markets because the chip company knew how to produce chips, not because the chip company had the best computer architectures.

Semiconductor Chips!

[Tutorial] - Quantum Memory: Superconducting qubits and Quantum Computer Hardware Design

Quantum Information and Quantum Communication

Quantum Communication advantages

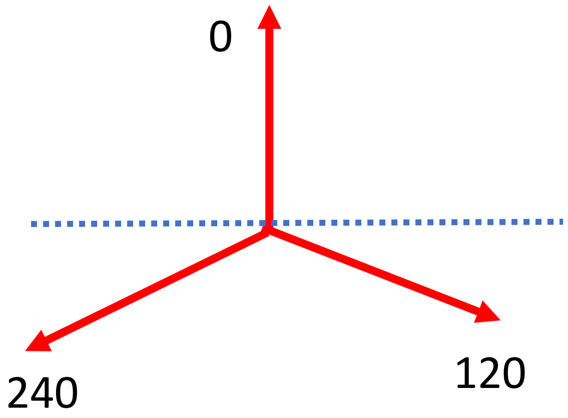
- Quantum Communications are two times faster than classical communication
  - Superdense Coding –Sending two bits of classical information through the transmission of single qubit.
- Quantum bits cannot be copies
  - No cloning theorem
- To intercept or measure quantum bits can be detected
  - Non-Disturbance

Teleportation:

- Background(History) – Asher Peres, Bill Wooters
  - Three(3) imperfectly distinguishable polarization states of a photon

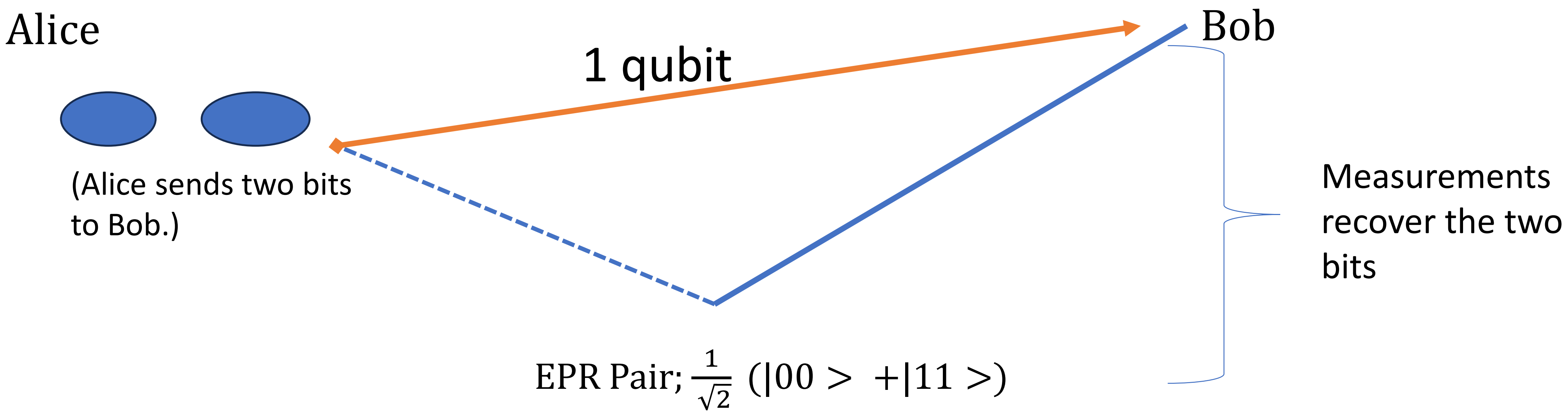
  $|\uparrow\uparrow>_{AB}$ or $|\searrow\searrow>_{AB}$ or $|\swarrow\swarrow>_{AB}$

  - More distinguishable in one Lab than two, joint measurement by having them both in the same lab, than by doing separate measurement.
  - You can only distinguish able two states, not 3-states
  - Entanglement helps
  - Charles Bennet(IBM) names "Teleportation"

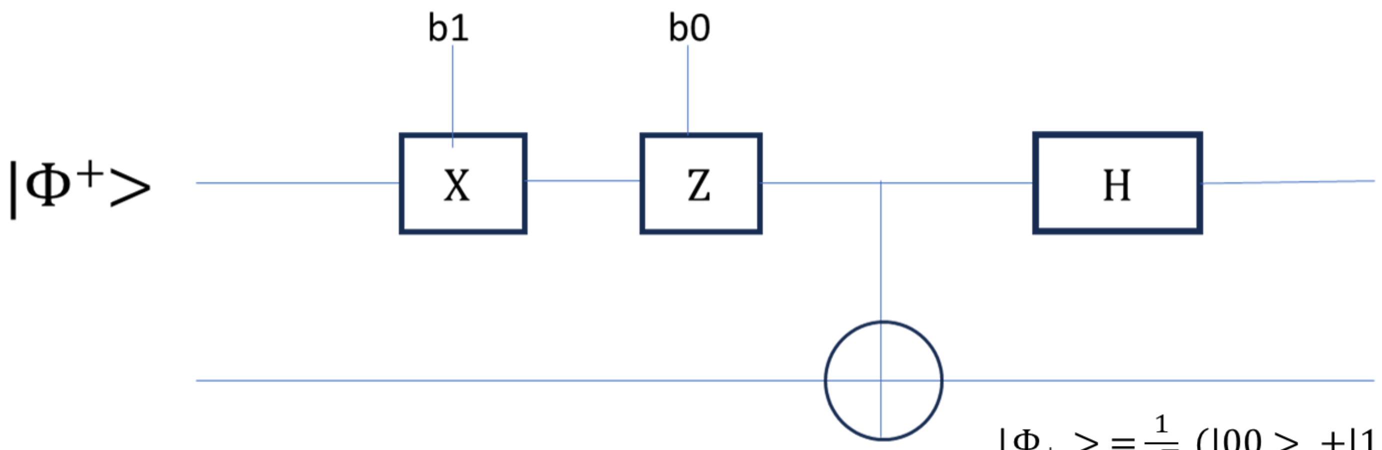


Superdense Coding

- quantum communications protocol
  - it can send two bits of classical information through the transmission of a single qubit.



Superdense Coding (2)



$|\Phi+\rangle = (1/\sqrt{2})(|00\rangle_{AB} + |11\rangle_{AB})$

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\Phi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

b0 b1

00	$|\Phi^+\rangle$
01	$|\Phi^-\rangle$
10	$|\Psi^+\rangle$
11	$|\Psi^-\rangle$

1. If Alice wants to send 00 to Bob, she only needs send her qubit to Bob, b0=0, b1=0. Identity operation
2. Alice wants to send 10 to Bob, she needs to apply X-gate.

$$(X\otimes I)|\Phi^+\rangle = |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

Quantum Communication:
Quantum Teleportation and Entanglement

- The No-cloning Theorem
  - One copy of unknow Quantum State, $|\psi>$, we cannot produce two copies of it.
  - If we can clone a Qubit, i.e. one copy of a Qubit $|\psi>$, we can make many copies, $|\psi>^{\otimes n}$
  - We need to show that the map,

$$|\psi>\otimes|\psi> \dashrightarrow |\psi>\otimes|\psi>$$ is not unitary

Quantum Information-- No-cloning Theorem

The No-cloning Theorem

- State $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$,if we can clone the a qubit, i.e. one copy of a qubit $|\psi\rangle$, we can make many copies $|\psi\rangle^{\otimes n}$
- One copy of unknown quantum state $|\psi\rangle$, you can not produce two copies of it.
- Give $|\psi\rangle|0\rangle$, To prove the no-cloning theorem, we need to show that the map $|\psi\rangle\otimes|\psi\rangle \rightarrow |\psi\rangle\otimes|\Psi\rangle$ is not unitary. Checking it does not preserve *Inner products.*
- Alice needs to do $|\psi\rangle\otimes|\phi\rangle \rightarrow |\psi\rangle\otimes|\psi\rangle$, i.e. Alice can make an unlimited number of copies of $|\psi\rangle$. Alice likes to take state $|\psi\rangle\otimes|\phi\rangle\otimes|\beta\rangle$ turns into $|\psi\rangle\otimes|\psi\rangle\otimes |\psi\rangle$

Example: No-cloning theorem

- Consider the gate, $U_x = I \otimes \sigma_x$ operating on the state |1>|0>

$$U_x|1> \otimes |0> = |1> \otimes |1> \quad \text{-- Eq. A}$$

Q: Does Eq. A violate no-cloning theorem?

A: No, It does not, why? Eq. A is True.

$$U_x|0> \otimes |0> = |0> \otimes |1> \;;$$

It does not copy the content of the second qubit into the first.

History Box:
1982 -- The no cloning theorem, published by Wooters, Zurek, and Dieks.

Example--Non-Cloning Theorem

- Let us define a new operator, U_{clone} can copy (clone) a qubit, i.e.

U_{clone} |0> → |0>|0>, U_{clone} |1> → |1>|1> (if it is true?)

Let us apply the U clone to superposition

$$U_{clone}\ (1/\sqrt{2})(|0> + |1>) \rightarrow (1/\sqrt{2})(|0>|0> + |1>|1>)$$

$$\neq [(1/\sqrt{2})(|0> + |1>)]\ [(1/\sqrt{2})(|0> + |1>)]$$

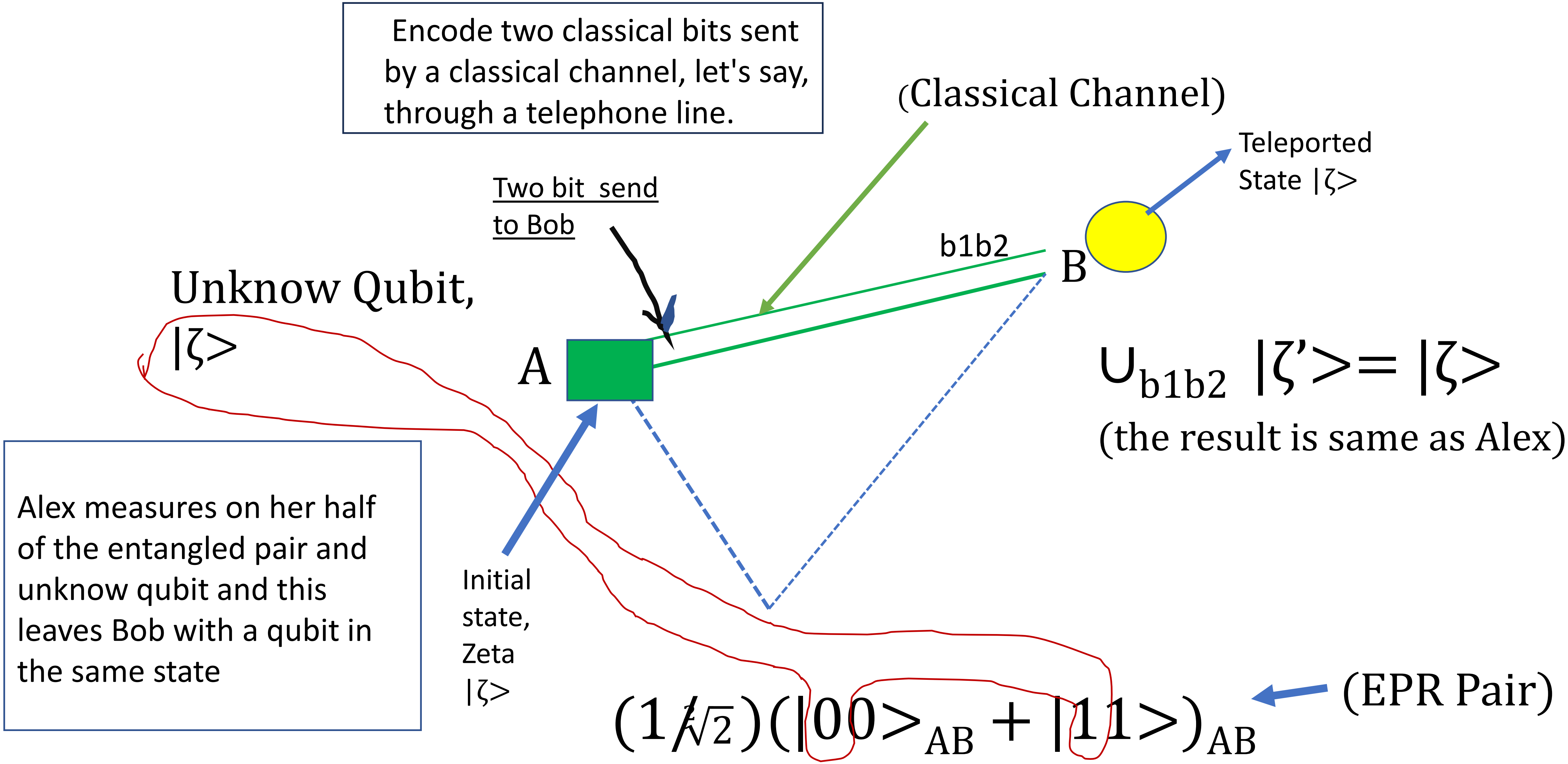
The nature of Quantum mechanics and linear algebra is that it is called the no-cloning theorem.

Quantum Teleportation (QT)

- Quantum Teleportation provides a mechanism of moving a Qubit from one location to another location, without having to physically transport the underlying particle to which that Qubit is normally attached.
- An important aspect Quantum information theory is " Entanglement"

History Box:

1993 – C. H. Bennett, G. Brassard, C. Crepeanu, R. Jozsa, W.K. Wooters
1994– Sandu Popescu, Anton Zeilinger(realized)
2017– Jian-wei PAN--- 870mile (1400Km) Longer distance, Micius Satellite


Encode two classical bits sent by a classical channel, let's say, through a telephone line.
(Classical Channel)
Teleported State |ζ>
Two bit send to Bob
b1b2
B
Unknow Qubit, |ζ>
A
U_{b1b2} |ζ'>= |ζ>
(the result is same as Alex)
Alex measures on her half of the entangled pair and unknow qubit and this leaves Bob with a qubit in the same state
Initial state, Zeta |ζ>
$(1/\sqrt{2})(|00>_{AB} + |11>)_{AB}$
(EPR Pair)

Quantum Teleportation (2) -- Notes

The two-bit send through Classical Channel can be duplicated. Alice and Bob has EPR pair and anyone (eavesdropper) can listen in on the conversation and the get the two bits on classical channel. But, if without the other half of the EPR pair. They cannot recreate the state, $|\zeta\rangle$. (Zeta)

Summary of the Quantum Teleportation Protocol:

"Quantum teleportation provides a 'disembodied' way to transfer quantum states from one object to another at a distance location, assisted by previously shared entangled state and a classical communication channel"
(Nature 518,526) (2015)

Quantum Teleportation (3) – Notes (protocol overview)

A. Alice makes a joint measurement on her bit of EPR pair state and her unknown state $|\zeta>$

B. She sends results to Bob.

C. The results let Bot perform a transformation on his half qubit of EPR Pair, which put it in the state $|\zeta>$.

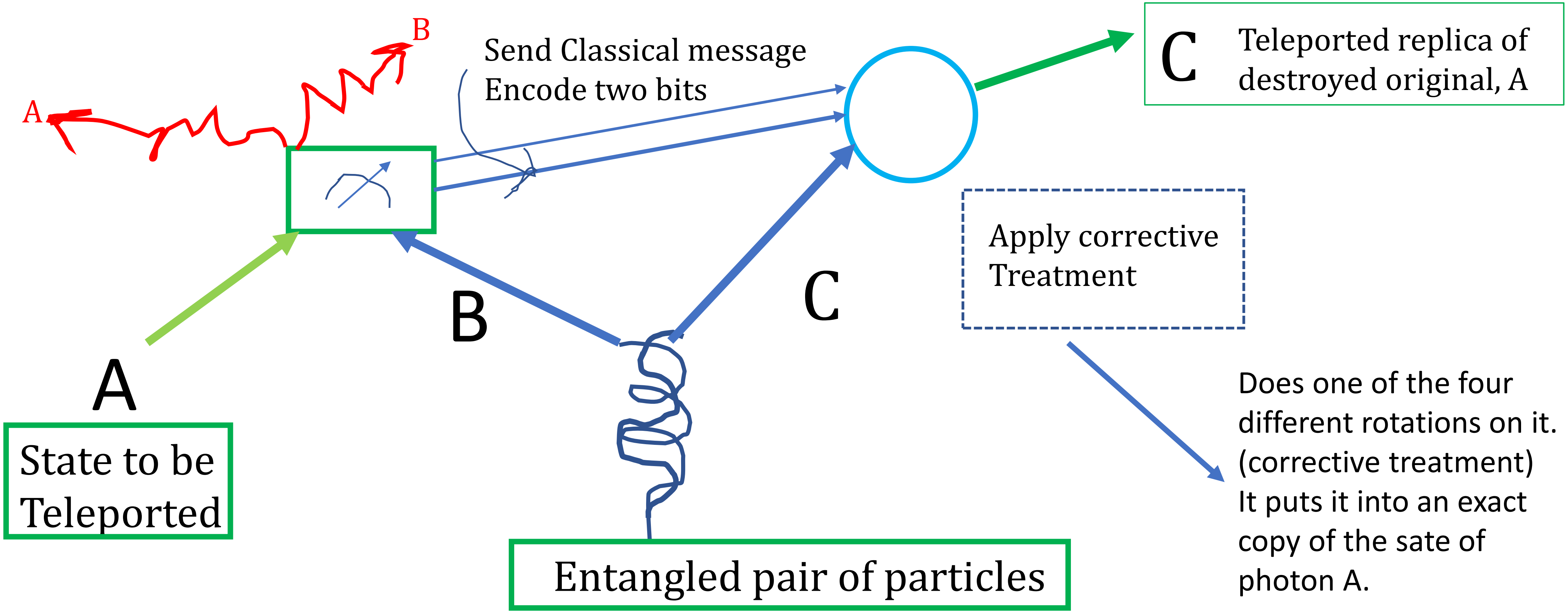
D. Alice: By measuring Qubit is state $|\zeta>$, Alice destroy its state, so the information in it is not in cloud.

E. Bob waits to receive the classical outcome of Alice's measurement, teleportation cannot transmit information faster than light.

(P. Shor's Class Notes)

Quantum teleportation and Entanglement

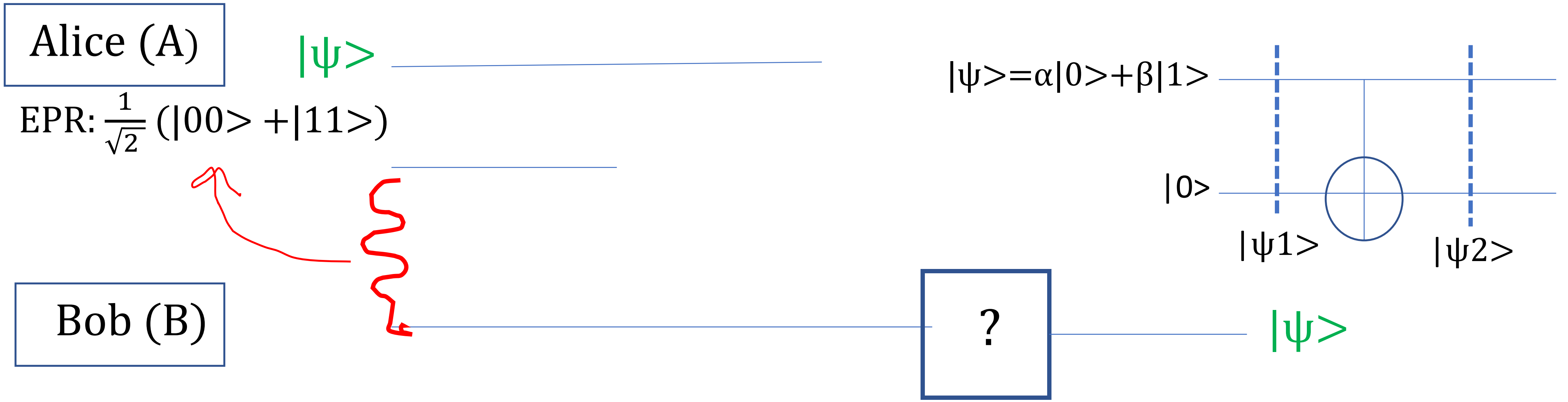
- Why it does not work without to use entanglement?



Quantum circuit– Teleportation

Quantum circuit:

1 e- bit (EPR Pair), 2-bit (classical bits)



$|\psi 1> = |\psi> \otimes |0> = \alpha|00> + \beta|10>$

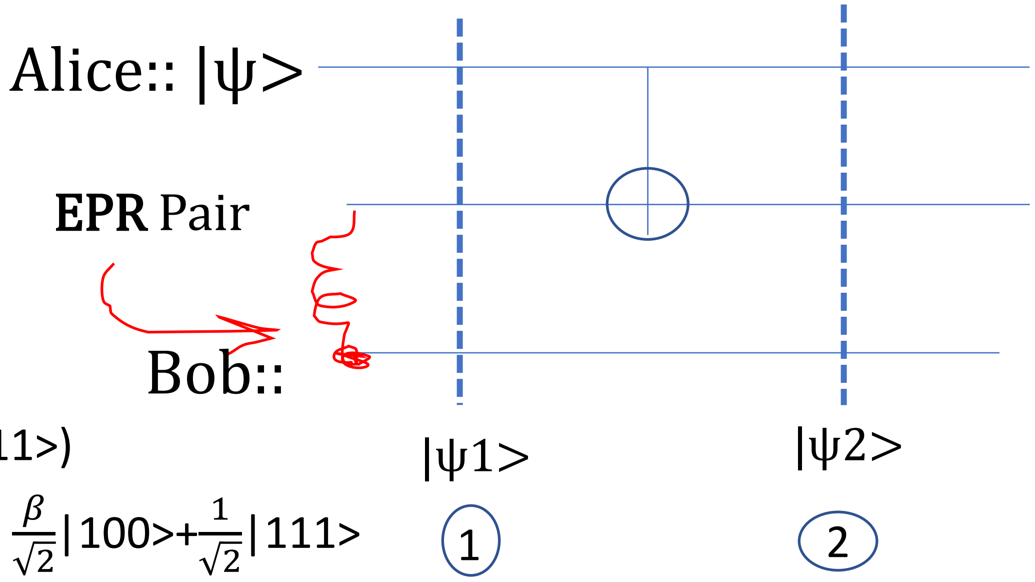
$|\psi 2> = \alpha|00> + \beta|11>$

A measure?

Quantum circuit– Teleportation

Quantum circuit 2

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



1. $|\psi 1\rangle = |\psi\rangle \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle)$

$$= \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|100\rangle + \frac{1}{\sqrt{2}}|111\rangle$$

Apply CNOT Gate:

2. $|\psi 2\rangle = \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|110\rangle + \frac{\beta}{\sqrt{2}}|101\rangle$; Alice measure her 2^{nd} bit of $|\psi 2\rangle$:(Target bit)

$\Pr.[0] = \left|\frac{\propto}{\sqrt{2}}\right|^2 + \left|\frac{\beta}{\sqrt{2}}\right|^2 = \frac{1}{2}$; State collapses, $\alpha|00\rangle + \beta|11\rangle \rightarrow |\psi\rangle$

$\Pr.[1] = \left|\frac{\propto}{\sqrt{2}}\right|^2 + \left|\frac{\beta}{\sqrt{2}}\right|^2 = \frac{1}{2}$; State collapses, $\alpha|01\rangle + \beta|10\rangle \rightarrow$ almost $|\psi\rangle$?

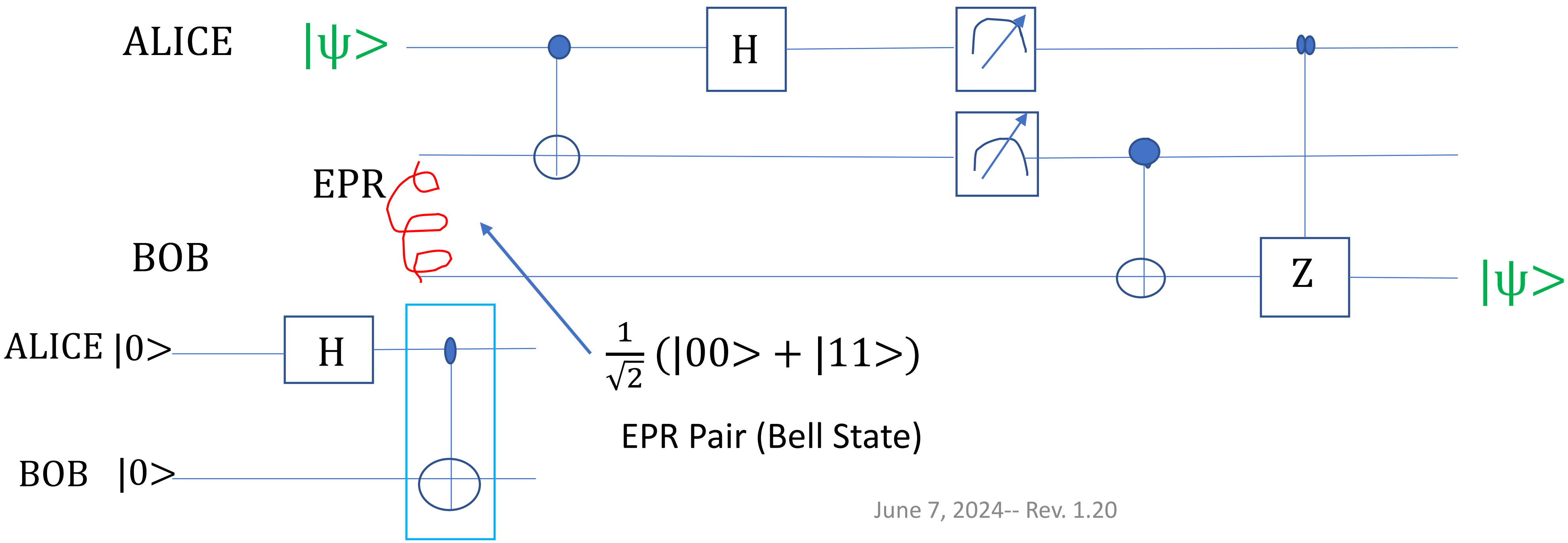
Quantum circuit– Teleportation

Quantum circuit 3

Alice phoned Bob her outcome, If “0” , Bob does nothing,

If “1”, Bob applies NOT gate to his qubit ,

Now, Bob and Alice share α|00> + β|11> = |ψ> state.



Appendix QT- Math Notes

(QC Teleportation Math Notes)

Quantum Key Distribution– BB84

- Quantum Key Distribution(QKD) is a protocol used to distribute shared secret Keys
- Security ideally based on Quantum Mechanics
- The BB84 protocol can be used to create a shared secret key for sender (Alice) and receiver (Bob).
- QKD is not Quantum Cryptography (not encryption)

- Inventors: Charles H. Bennet—IBM

  Gilles Brassard– University of Montreal

  1984.

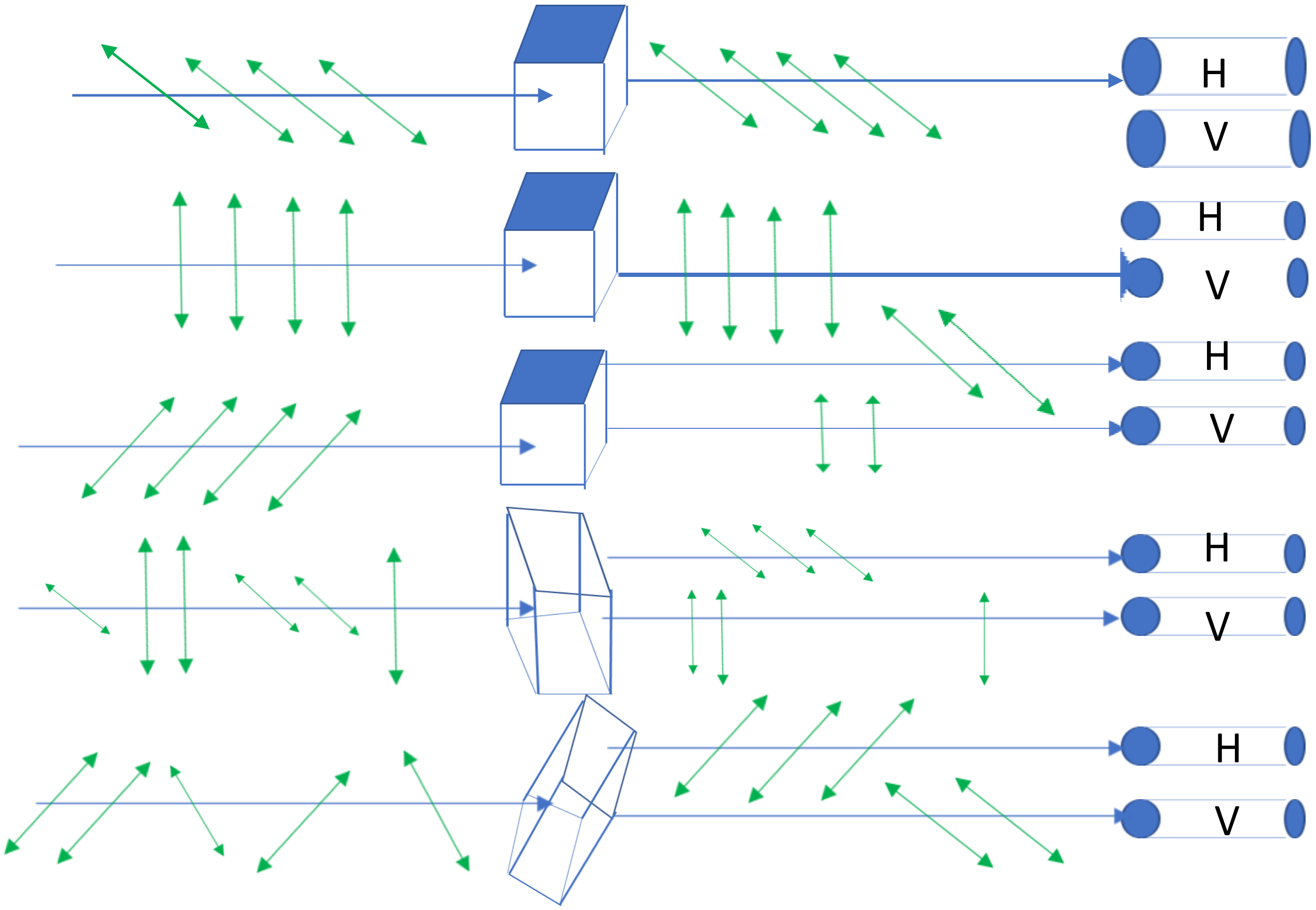
Quantum Key Distribution– BB84
--Using Polarized Photons to carry information

Horizontal Photons

Vertical Photons

Θ polarized Photon

Rotating apparatus

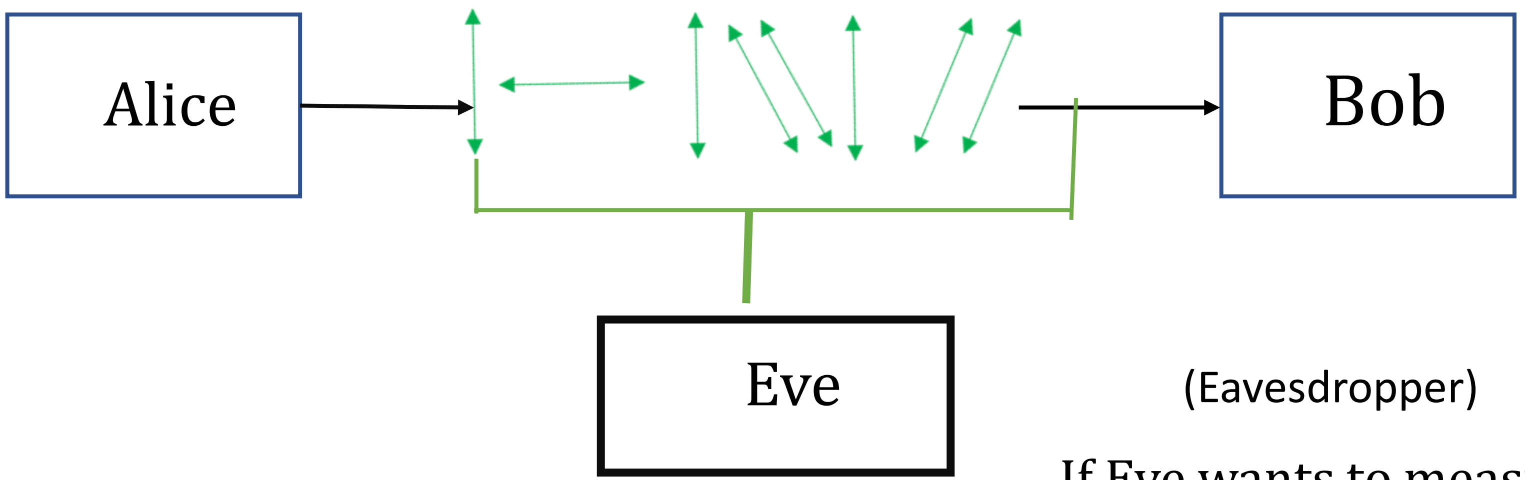


Probability $\cos^2 \theta$

Probability $\sin^2 \theta$

1.Distinguish a left diagonal from a right diagonal
2.Rotated apparatus cannot distinguish H/V

Quantum Key Distribution– BB84
--Using Polarized Photons to carry information (2)



(Eavesdropper)

If Eve wants to measure the photons, she will introduce errors with high probability.

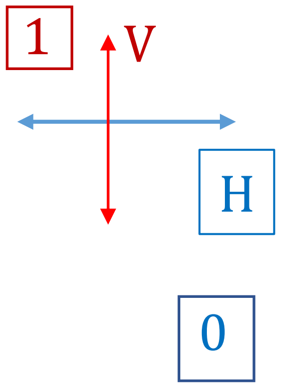
BB84 Protocol

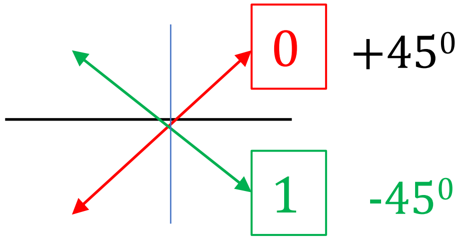
Horizontal and Vertical Photon Polarization

|H>, |V>

45^0(counter-clockwise), 45^0 (clockwise)

|+45> (Right-diagonal), |-45^0> (Left-diagonal)





Basis	0	1
+	↑	⟶
×	↗	↘

Quantum Key Distribution– BB84
--Example

	Photon 1	Photon 2	Photon3	Photon4	Photon5	Photon6	Photon7	Photon8	
Basis/Alice's Random sending	+	+	×	+	×	×	×	+	
Photon polarization Alice's sends	↑	→	↘	↑	↘	↗	↗	→	
Bob's random measuring basis	+	×	×	×	+	×	+	+	
Photon Polarization Bob Measures	↑	↗	↘	↗	→	↗	→	→	
Shard Key	0		1			0		1	

Entanglement Based Protocol

- BBM92, E91 protocols are Entangled states (Maximally entangled-Bell state)

  Entangled source→ two-photon

a. One Photon sends to Alice, one photon sends to Bob. The photons are sent through Quantum channels → reliable transmit single-photon (One Qubit only) and preserve their entanglement, Bell State.

b. Photons are correlated in measurement basis: Horizontal and vertical photon polarization and 45^0 degrees 135^0 degrees diagonal

c. Alice and Bob choose the same measurement basis →measurements results are correlated between the measurements.

Entanglement Based Protocol—BBM92 (2)

- C-1. If Alice is H/V and Bob is $45^0/45^0$ measurement → Not Correlated
- C-2. Intermediate basis choices → Partially correlated
- C-3. Alice/Bob uses H/V or $45^0/45^0$ basis record the measurement's results(many times)
- C-4. The remaining protocol steps are the same as BB84.
  - Use classical channel measurements and uses the same basis
  - Compares the results, error rate below 20%.
  - Classical post processing to erase any information that a potential eavesdropper could receive it.

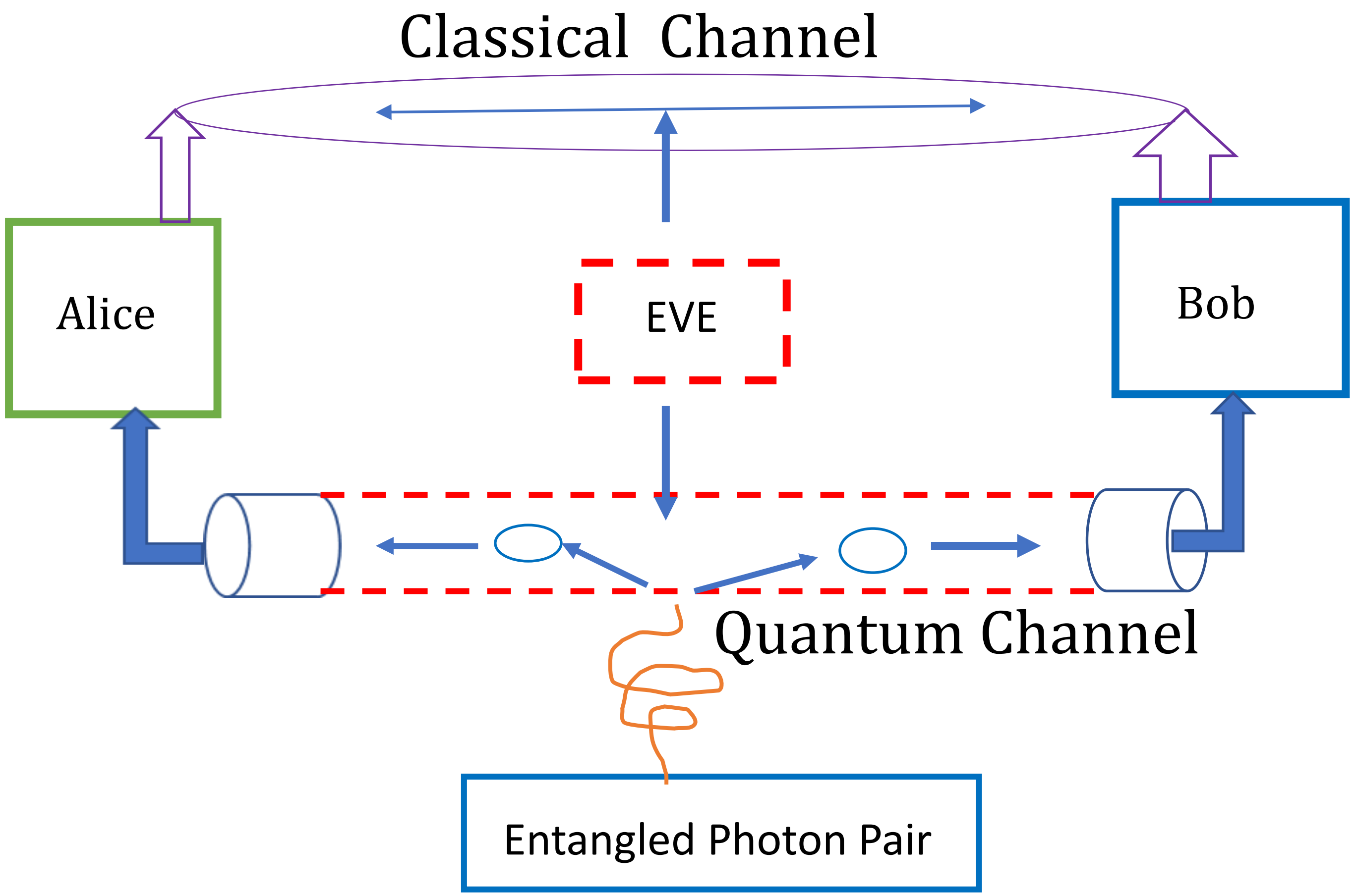
Benefit of BBM92:
Does not require trusted source to prepare the Send/Receive State.

BBM92- Example

	Alice/ Bob	Alice / Bob	Alice / Bob	Alice / Bob	Alice/ Bob	Alice/ Bob	Alice/ Bob	Alice/ Bob	Alice Bob	
Receiving Basis	× +	× ×	+ ×	+ +	× ×	+ ×	+ +	× +	× ×	
Measurement	↗ ↕	↖ ↗	← ↗	↕ ↔	↗ ↖	↕ ↖	↔ ↕	↗ ↔	↗ ↖	
Convert to bit →↗ |0> ↑↖ |1>	0 1	1 0	0 0	1 0	0 1	1 1	0 1	0 0	0 1	
Sifting Same Basis?	No	Yes	No	Yes	Yes	No	Yes	No	Yes	
Inversion Bob inverts his bits		1 1		1 1	0 0		0 0		0 0	
Security Test for errors?		Yes		No	No		No		Yes	
Final Key Kit bit generated				1	0		0			

Source: Chris Erven/U. of Waterloo, Canada

Entanglement Based Protocol -E91 (Eckert 91)–
Eckert, Chao and Lo



Entanglement Based Protocol -E91

- E91 (Ekert91) is about the same as BBM92 with extra
- Alice and Bob randomly chooses between one of the three measurement basis-- Horizontal/Vertical basis, 45 degree rotated diagonal/antidiagonal basis, and intermediate basis (22½ degrees)/22.5 degrees.
- As BB84, Alice and Bod discussed their measurements basis through classical channel.
- Alice interprets H, D states as 0 and V, A states as 1. Bob should do the opposite to get the same key if the state is used. In this case they will have the identical key.

Summary- Entanglement Based Protocol -E91

- Ekert91 requires entanglement, (BB84 does not require entanglement) but it does not require Bell state measurement.
- Ekert91 requires two parts to implement the protocol successfully.
- Alice and Bob detect eavesdropper by calculating a pre-defined correlation function using the photons they measured in different basis.

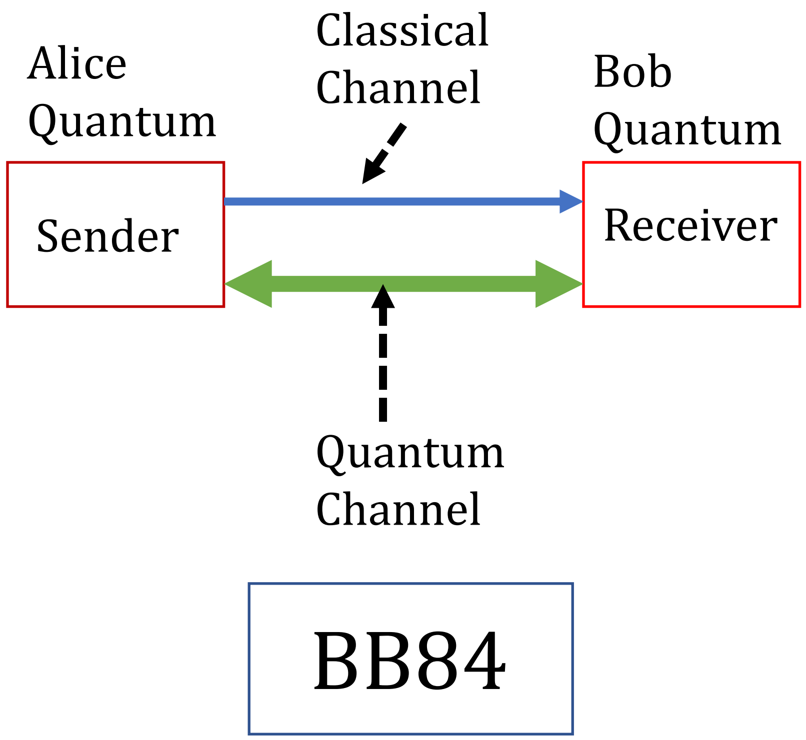
QKD-- Summary

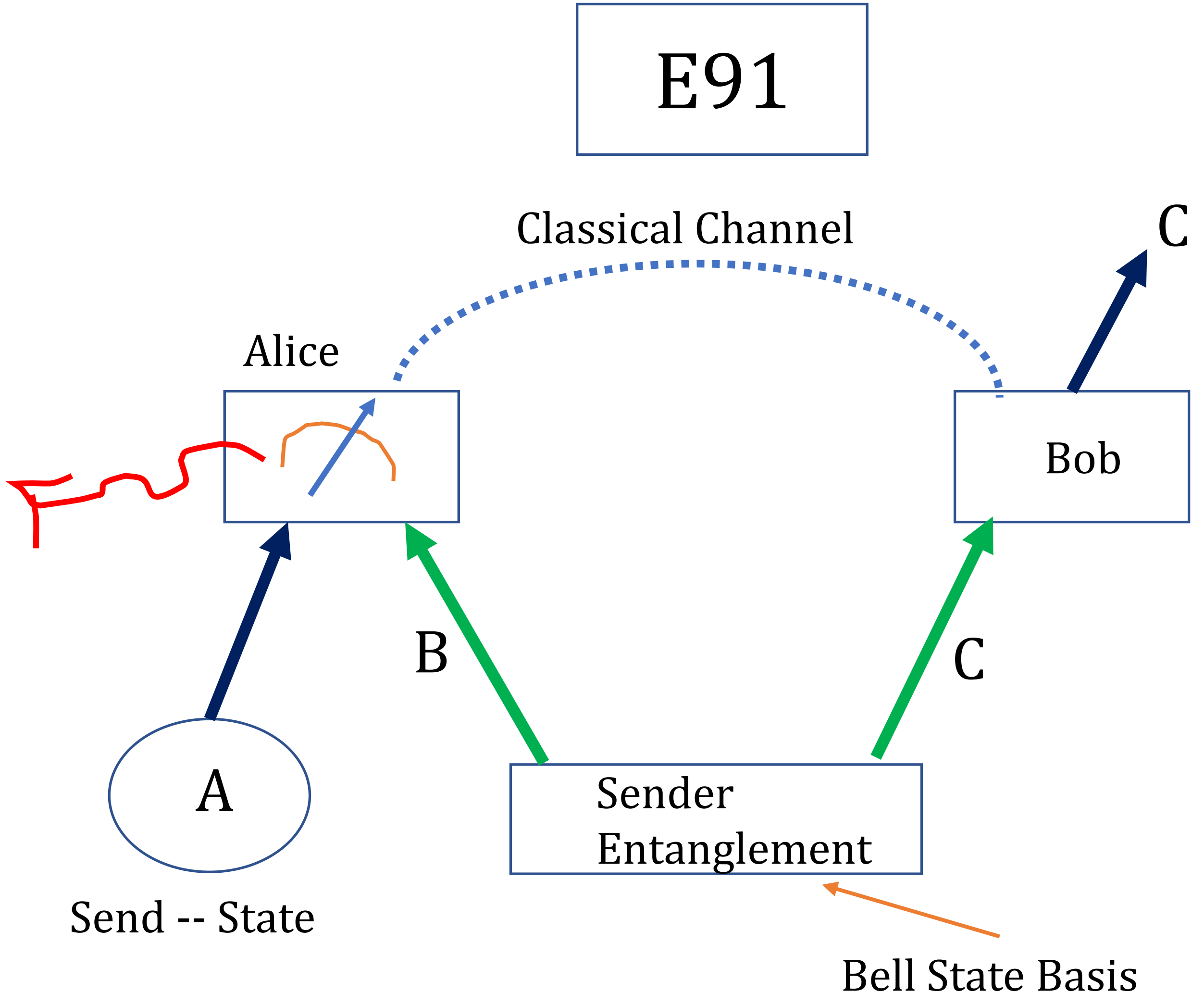
- 1. Many forms of QKD protocols are using Quantum Mechanical properties
- 2. BB84 QKD (Quantum Key Distribution)
  - Alice ---(prepare)—the states ----(send) ---→ Bob
  - Bob measures the STATES.
  - Bob and Alice → comparing the preparation basis and measurement basis
  - Follow several classical steps to establish secure Encryption Keys(Shared keys).
  - Quantum superposition prepares states (see Ex.)
  - Prepare and measure QKD protocols

QKD Types

QKD types	Protocols	Quantum Physical Property	Models
Prepare and Measure	BB84	Superposition	BB84 Model
Entanglement Base	BBM92 E91	Entanglement	E91 Model

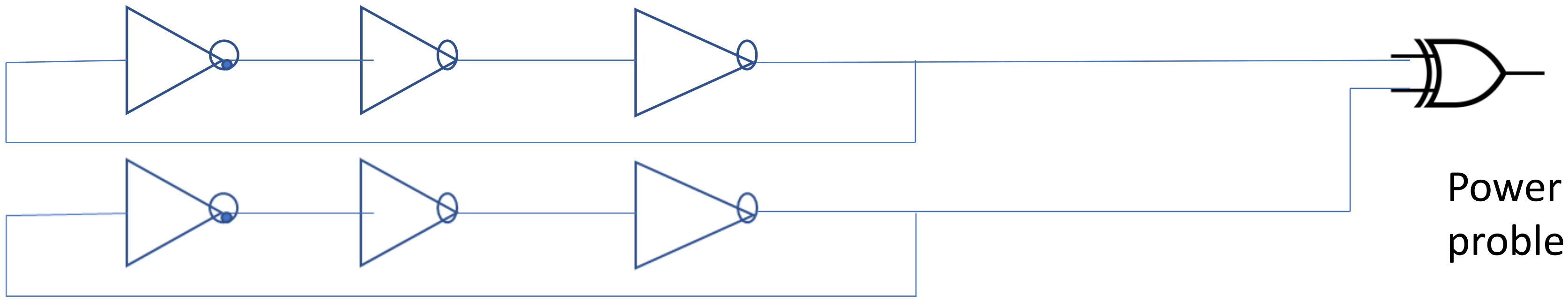
BB84 and E91 Models





Random Numbers generator

- Public key encryption and other cryptographic tasks require randomness.
  - ATM, digital signature, Monte Carlo simulation, and lotteries
  - Need Random number generator (RNG)
- Pseudo random generator– classical (standard) computer
- Hardware random generator or single photon hitting a beam spitter



Power noise can cause problem, not random

Quantum Repeaters

- Long distance connection
  - Photon loss in optical channel, in classical communication (to compensate the loss), uses amplifiers as repeater.
  - No-cloning theorem prohibits the amplification of an unknown quantum state. [cannot repeat]
- Short wave lengths- elastic scattering change the propagation direction of light, escape the fiber.
- Long wave lengths- absorption by the material itself
- Loss – 0.2bB/Km (attenuation coefficient)
- Transmitted power scales very poorly with distance, 500Km (100bB loss)

Quantum Repeaters (2)

- Quantum entanglement is key resource needed to realize Quantum Repeater
- A distributed entangled state can be used as a resource for Quantum repeater
- Loss of information– scattering and absorption of photons during transmission over optical fibers.

Quantum Algorithms

Deutsch-Jozsa Problem

Shor's Algorithm

Appendix 3: Deutsch-Josa Problem Math Notes

the Future of Memory and Storage

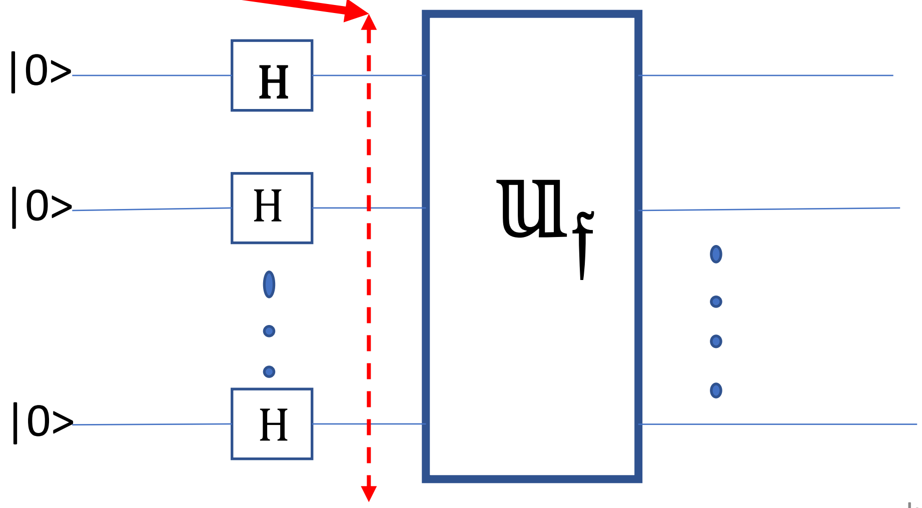
General Computational Process:

- Quantum Circuit Model
  1. Massive Superposition state to SET the stage for Quantum Parallelism and Quantum Interference during the Algorithm in action.
  2. Equal Superposition State: 3-Bit Qubit, initialized in |000>
  3. Apply Hadamard Gates,

$H \otimes H \otimes H|000\rangle = \frac{1}{\sqrt{2^3}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$

Massive Superposition

$H^{\otimes n}|0\rangle_n = \frac{1}{\sqrt{2^n}} \sum_{0 \le \chi \le 2^n} |\chi\rangle_n$; n-Qubit state $|0\rangle_n$, ;where $H^{\otimes n} = H \otimes H \otimes \ldots\ldots \otimes H$, n-times



Define, U_f on the computational-basis states,

$|\chi\rangle_n |y\rangle_m$; n=input register, m=output register

$U_f (|\chi\rangle_n |y\rangle_m) = |\chi_n\rangle |y \oplus f(\chi)\rangle_m$

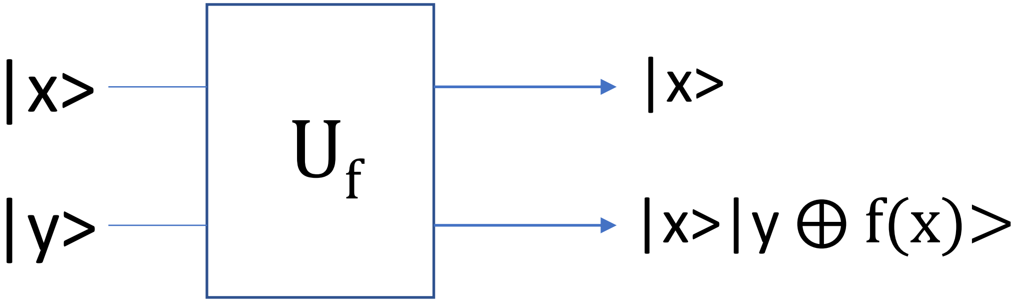
$\oplus$ modulo -2 bitwise addition, bitwise XOR,

$1101 \oplus 0111 = 1010$

Implement a Universal Quantum Algorithm

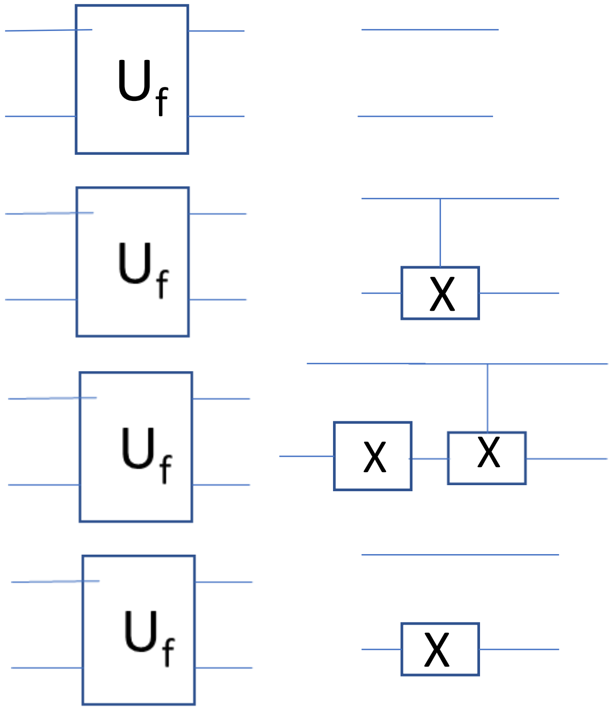
- Suppose we are given a block box

$$\rightarrow U_f\,(|x\rangle|y\rangle) = |x\rangle|y \oplus f(x)\rangle$$



	x=0	x=1
f_0	0	0
f_1	0	1
f_2	1	0
f_3	1	1

The four distinct functions $f_j(x)$ that take one bit into one bit



f(0)	f(1)
0	0
0	1
1	0
1	1

Deutsch-Jozsa Problem

- Problem definition: Find out f(X) is Constant or Balanced.

$X_1, X_2, \ldots\ldots X_N \rightarrow$ F(X) — $f(X_1, X_2, \ldots. X_N)$

N-bits

"0' or "1"
(single result)

f(X) is either Constant or Balanced

Constant:

All $f(X_1, X_2, \ldots. X_N) = 0$
or
All $f(X_1, X_2, \ldots X_N) = 1$

Balanced:

Half are 0
And
Half are !

Deutsch-Jozsa Problem(2)

- Classical/Query # $2^N/2$, (always works)
- X=(1,1,.....0)- f(X) —— f (1,1,....0) = f ($X^{(i)}$) ={0,0....0, ?.....}

($2^N/2 = 2^{N-1}$)

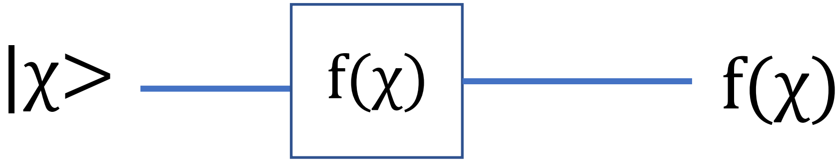
($2^N/2 + 1$)

Classical : requires $2^N/2$ +1 steps

Quantum: 1 –step, always works.

- Quantum gives an Exponential Speed Up: 1 vs. 2^{N-1} +1.

Quantum Circuit: Example (1-bit)-- The Deutsch – Jozsa Problem



Constant : f(0) = f(1) = 0, or 1

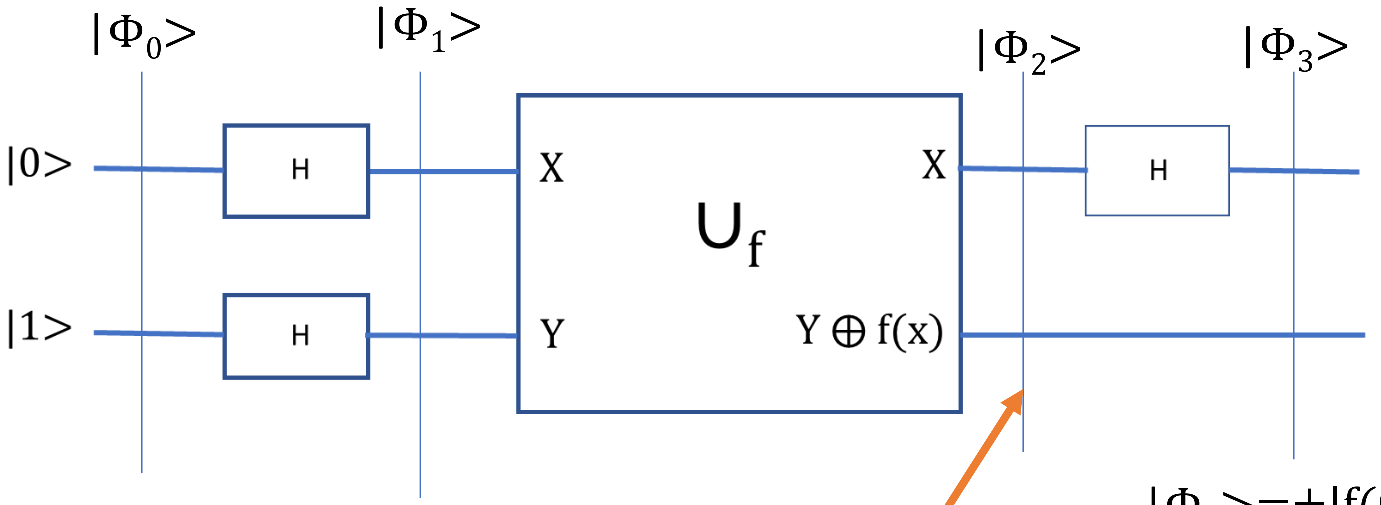
Balanced: Exactly Half are “0” or Exactly Half are “1”

Type	f(x=0)	f(x =1)	f(1) ⊕ f(0)
Constant	0	0	0
Constant	1	1	0
Balanced	0	1	1
Balanced	1	0	1

Classical: Requires $\frac{2^N}{2} + 1$ steps, N=1 → 2 steps

Quantum : 1- step, always works.

Deutsch-Jozsa Problem—Quantum Circuit



$|\Phi_3> = \pm|f(0) \oplus f(1)>[(0> + |1>)/\sqrt{2}\ ]$

$|\Phi_0> = |0> \otimes |1>$
$= |01>$

$|\Phi_1> = H \otimes |01>$

Examine $y \oplus f(x)$
f(X) =0, then $y \oplus f(X) = y \oplus 0$
$f(X) = 1$, then $y \oplus f(X) = y \oplus 1$

$|\Phi_2> = \pm 1/\sqrt{2}\,|0>(|0> - |1>)$,if f(0) = f(1)

or

$|\Phi_2> = \pm 1/\sqrt{2}\ |1>(|0> - |1>$, if f(0) $\neq$ f(1)

The Deutsch – Jozsa Problem
1-bit example(requires two-Qubit)

- Quantum Circuit
  - 1-bit example(requires two Qubits)
- Math Notes (See Appendix ~~7-1~~) Appendix 3-- Math Notes

Shor's Algorithm

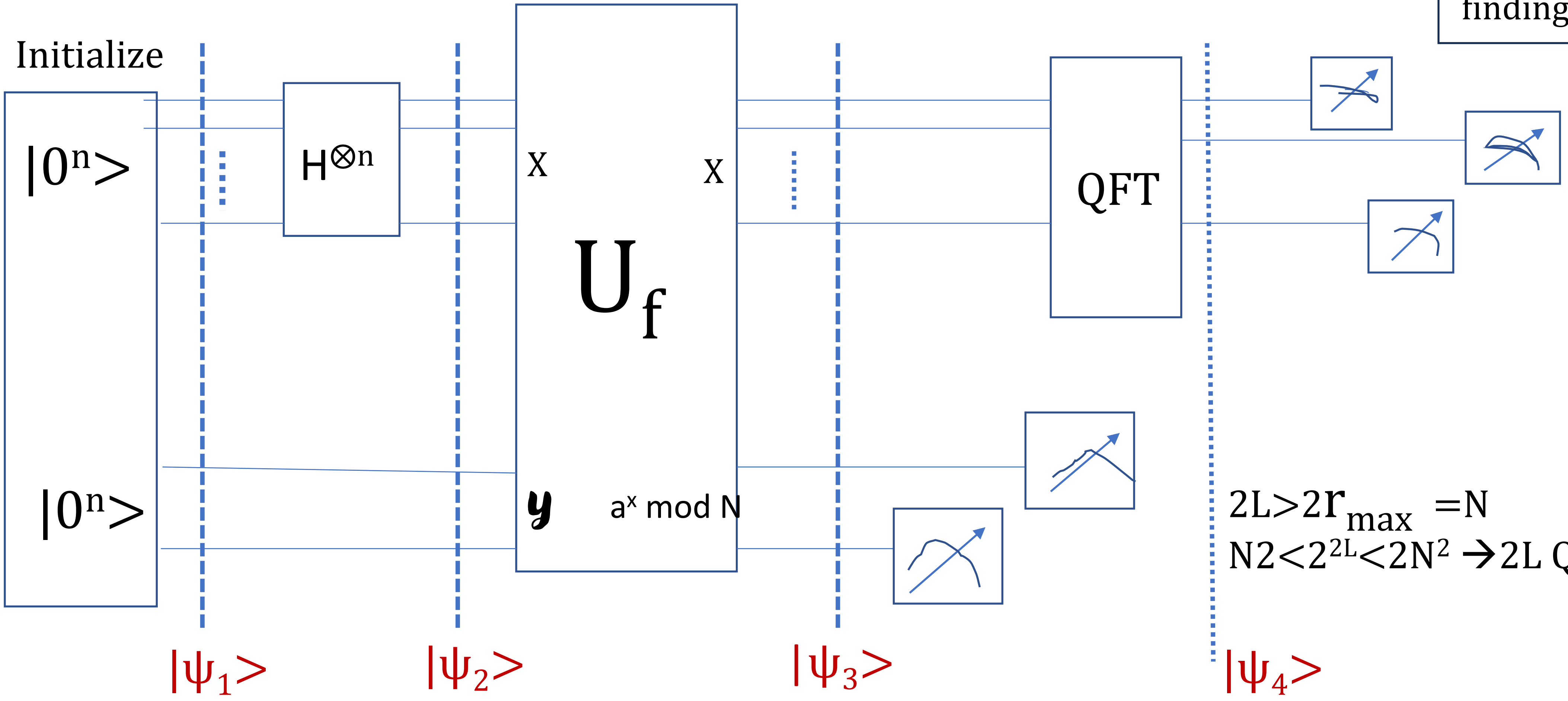
- Factor numbers into two Prime, $O(n^3)$ i-- Quantum Operations; for best Classical Algorithms are exponential.
- Factoring is a hard problem
- Shor's Algorithm being able to factor in polynomial time

Shor’s order(period) finding algorithms and factoring

- Quantum Fourier Transformation:
  - For extract the periodic component in functions
  - Finding the period of a modular exponential function, aka “Order Finding”
  - Shor’s algorithm is to Factor Large Number, N.
  - Shor’s algorithm for order-finding is combined with a classical computation steps. Polynomial in input size, $n=\log_2 N$ scaling as $O(n^2 \log n \log\log n)$, the Quantum part of Shor’s algorithm is called “Period Finding”
  - Classical algorithm (Number filed Sieve) super polynomials i.e. exp $(O(n^{1/3}(\log)^{2/3}))$
  - The period finding algorithm determines the period r of a periodic function, $f(x) = f(x+r)$ from Modular Expontiation

Finding Period -- Quantum Circuit (Example)

Shor's algorithm is called "Period finding"



$2L > 2r_{max} = N$

$N2 < 2^{2L} < 2N^2 \rightarrow 2L$ Qubits

$|\psi_1 = H^{\otimes n}|0^n> = 1/\sqrt{2}^n \sum_{x=0}^{n-1} |X>$

$|\psi_2> = (1/\sqrt{2^n}) \sum_{x=0}^{2^n-1} |X>|f(X)>$

Steps of the Quantum Order Finding Algorithm

- Step 1: $(2^{r_{max}} > 2r_{max})$ $2^L > 2\, r_{max}$

  $|\psi_1 >= |+>^{2L}|0> = 1/2^L \ \sum_{X=0}^{2^{2L}} |X >$
- Step 2: Compute F

  $1/2L \ \sum_{x=0}^{2^{2L}} |X > |f(X) >$
- Step 3: Apply QFT on the first register

  $1/2^L \ \sum_{z=0}^{2^{2L}} \ \ \sum_{z=0}^{2^{2L-1}} |Z > |f(X) > e^{-2\pi i XZ/22^L}$
- Step 4: Measure f(X), X_0 is the smallest X
- $\sum_{z=0}^{2^{2L}} \ \sum_{y=0}^{2^{2L}/r} |f(X_0) > |Z > e^{-2\pi i(X_0+yr)Z/2^{2L}}$

$|x, t> \rightarrow |x, t \oplus f(x)>$

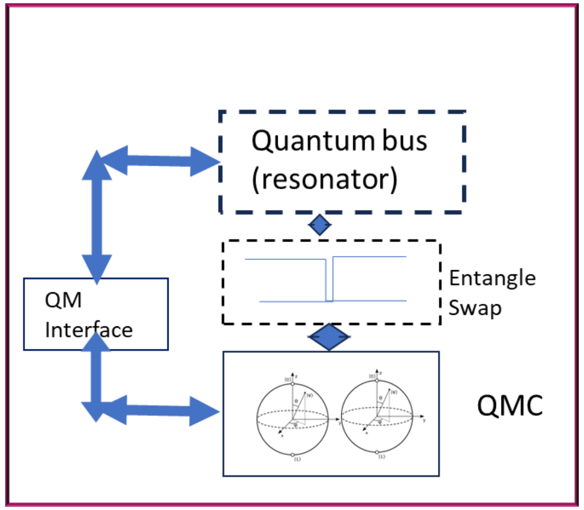
Geometric Sum

	Classical	Quantum	Quantum advantage	
Fourier Transformation	$O(n2^n)$	$O(n^2)$ $O(nlogn)$[1] gates	Quadratic polynomial in the number of qubits Exponentially speed	N= number of qubits or classical bits
Deutsch-Jozsa Problem	$2^{N-1}+1$ (steps)	1- step	Exponentially speed	
Simon problem	at least $\Omega(2^{n/2})$ queries	O(n) queries to the black box	Exponentially speed	
Shor's Algorithm[2]	$O\left(e^{-1.9(logN)^{1/3}(loglogN)^{2/3}}\right)$ ------------------------ $\sim e^{(\log N)^{1/3}}$	$O((\log N)^2 (\log\log N)(\log\log\log N))$ ------------------------ $\sim(\log N)^3$	Polynomial-time got integer factorization	To factor an integer N
Grover Search Algorithm (1996)	O (N)	$O(\sqrt{N})$		

[1] :Late of 2000, the best Quantum Fourier transform algorithm, Wikipedia,
[1a]: Quantum Fourier Transformation discovered by Don Coppersmith, 1994.
[2]: Classical resource is memory and runtime, Quantum resource is physical bits and runtime.

[Tutorial] - Quantum Memory: Superconducting qubits and Quantum Computer Hardware Design

Quantum memory and quantum qubits

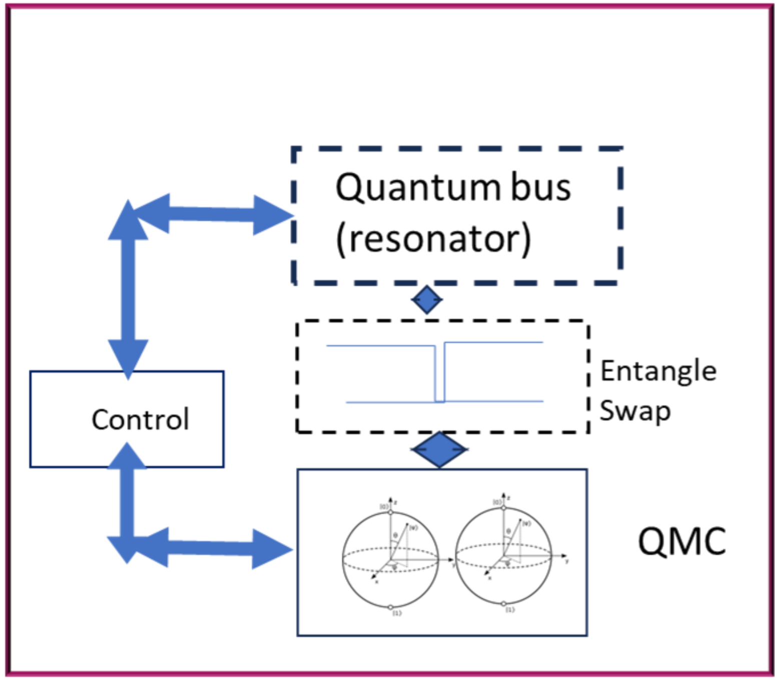


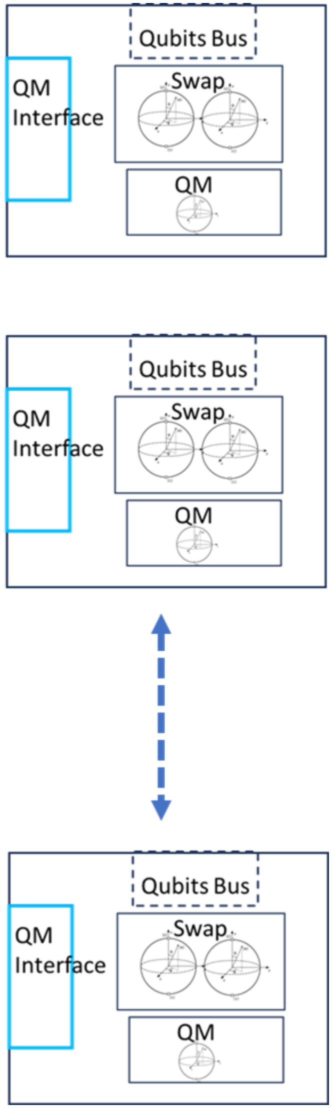
Quantum memory and quantum qubits

- Elementary element of a quantum memory device;
  Quantum memory cell– qubits or qubits array (register)
- Quantum computing harnesses the principles of quantum mechanics to perform computation
- Quantum computer perform computation exponentially faster than classical computer.
- Complex problems: Cryptography, optimization, quantum chemistry, and finance
- Quantum information across long distances, facilitating secure network– Quantum teleportation, Quantum Key Distribution(QKD)

Quantum Memory Cell

- QMC is made using Quantum register or a Qubit
  - Entangling gate between QMC are not necessary
- QMC consists three functions: 1. store quantum information. 2. controlled operation to store quantum information, 3. load information from QMC.
- Quantum memory requirements
  - Storing quantum information for a extended time
  - Fast and reliable RW operations
  - Ability to scale up to large quantum memory devices





Quantum Memory scale Up:

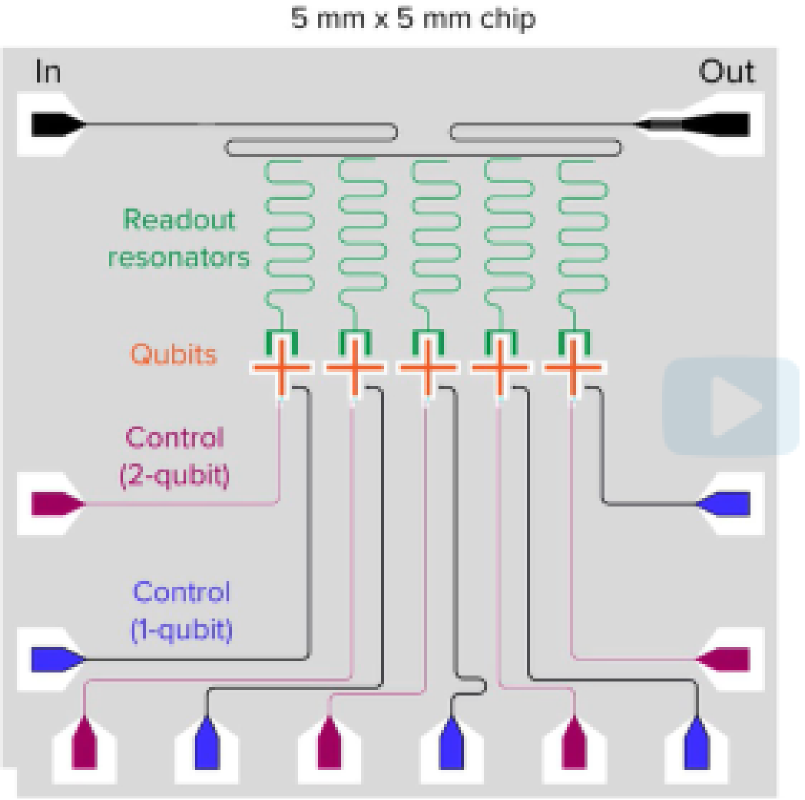
- How to scale up?
- Fault-tolerant large-scale quantum computer
  - IBM-Q: Eagle device with127 physical qubits
  - Osprey device with 433 qubit
  - Google Sycamore : 54 qubits
  - Quantinuum : 32 trapped ion qubits
  - IonQ: 20 qubits
- Scale up challenges: Integration of large numbers of qubits, large die size and cross talk
- Require a large number of SWAP gates to perform a gate between two qubits

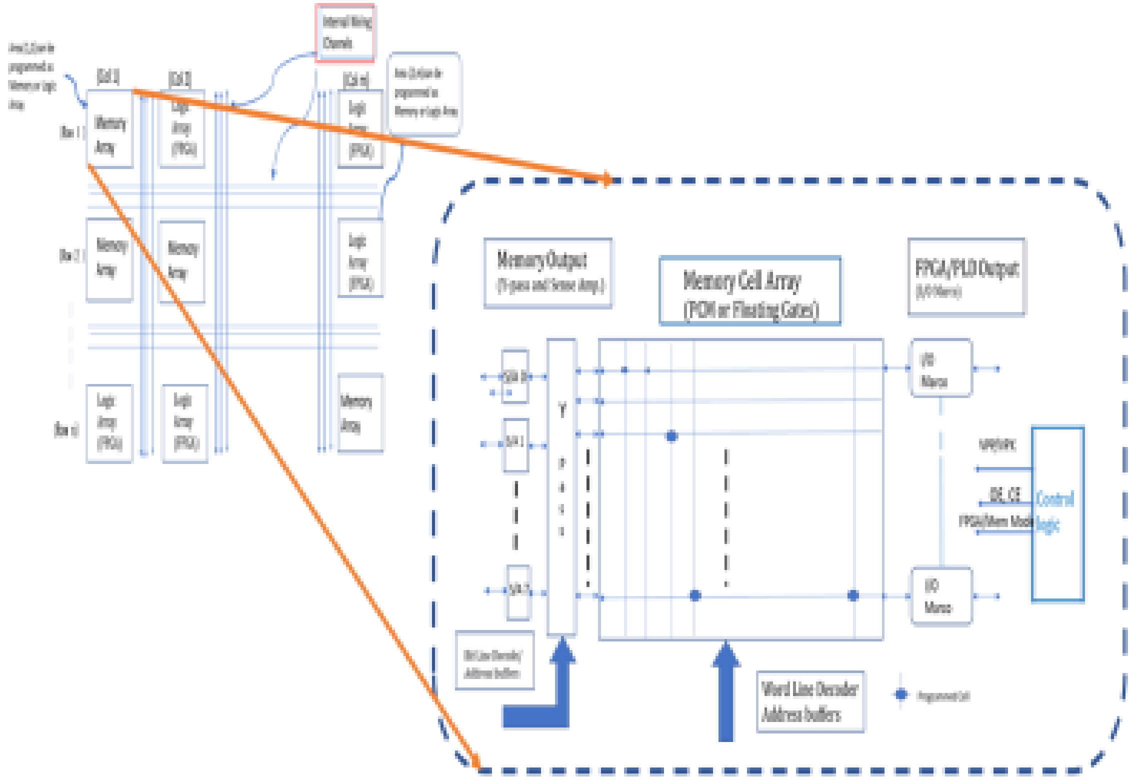
Quantum Memory vs Classical Memory

- Quantum no-cloning theorem
  - Reading and writing operations cannot copy information
  - All operations can relegalized using quantum operations, such as entanglement, SWAP gats
  - SWAP gates as RW operations, the reading and writing process can be completed with one SWAP gate, unlike classical memory we need a register
  - Quantum memory array cannot perform signal multiplexing as classical memory array

Qubit Arrays vs Classical Memory Arrays (2)

- Qubit Arrays
  - Requires RF pulses in the 4 to 6 GHz to manipulate their states
  - Control line for every Qubit at room temperature (RT) to 10mK(-273.14 ^{0}C)
  - Control and Read-Out circuits have to access each Qubits from RT to low temperatures.
  - Each Qubit is coupled by Resonator
- Classical Memory Arrays
  - Memory cells form as a Matrix, peripheral circuits, X-Decoder, Y-Decoder to access the selected cells. No induvial control lines are required.





Qubit cell array

Classical Memory cell array

Source: MIT

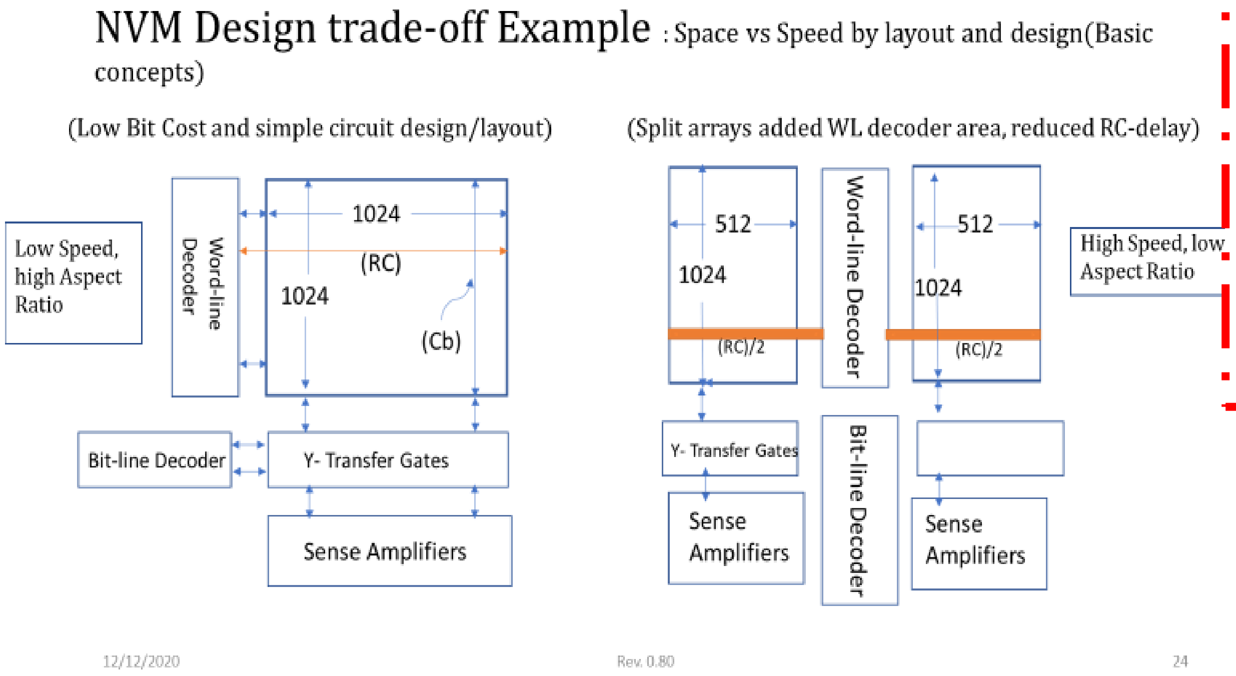
Quantum Technoloyg, LLC --- Rev 0.10

Source: W. Liu

15

Qubit Array vs Classical Memory Array

5 lines to access one Qubit



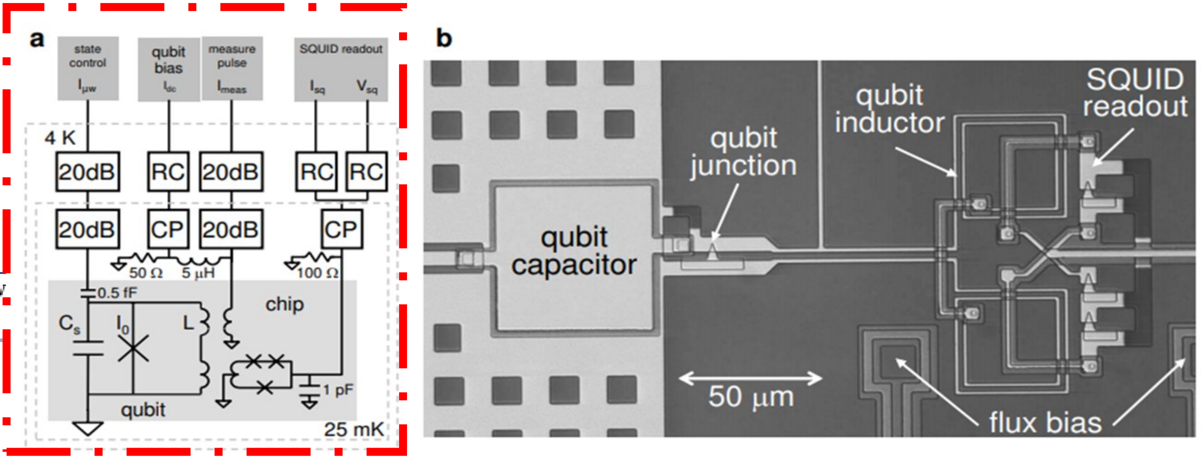


Fig. 2 (a) Schematic diagram of a phase qubit circuit and bias lines. Symbols 20 dB, RC, and CP represent 20 dB microwave attenuators, resistor-capacitor low-pass filters, and copper-powder microwave filters, respectively. The RC filters for the qubit and SQUID bias has 1 kΩ and 10 kΩ DC resistance, respectively, and a roll-off frequency ~ 5 MHz. The 5 μH inductor is a custom made radio-frequency bias tee with no transmitting resonances below 1 GHz. (b) Photomicrograph of present phase qubit, showing small area (~ 1 μm^2) junction shunted by a parallel plate capacitor. Microwave drive line (with capacitor, not shown) comes from the left. The qubit inductor is coupled to a SQUID readout circuit in a gradiometer geometry. The flux bias lines for the qubit are symmetrically placed about the SQUID and counter-wound to inhibit flux coupling to the SQUID. The SQUID bias line exits to the right. The holes in the ground plane inhibit trapped vortices in the superconducting ground plane.

Quantum Memory vs Classical Memory

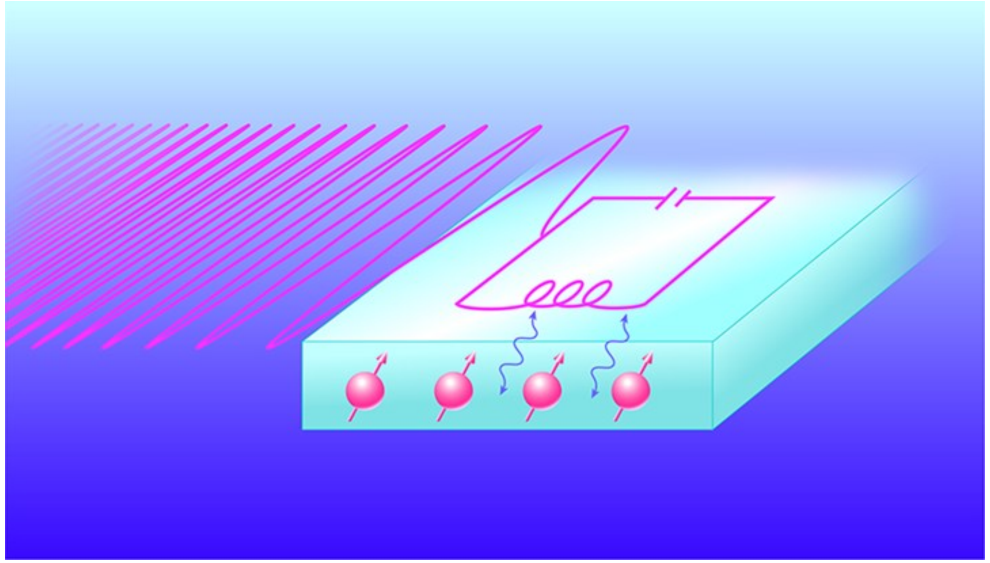
Classical computer vs Quantum computer:

- Classical computer
  - CPU ⟷ Memory :: Load data from memory and store data to memory
- Quantum Computer
  - QPU ⟷ Quantum Memory :: Load Quantum states from the memory

Reference: Chenxu Liu, et el. ; Quantum Memory: A Missing Piece in Quantum Computing Units arXiv:2309.14432v2, 2 Nov 2023

Quantum random-access memory device

- Chirped electromagnetic pulse, a superconducting resonator, significantly more hardware-efficient devices
- RAM- Write and Read, is an important part of a computer, a memory bank
- Quantum RAM speeds up the execution of a quantum algorithm.
  - Chirped microwave pulses to store and retrieve quantum information in atomic spin.



Superconducting circuit resonator and a silicon chip embedded with bismuth atoms. Chirped microwave pulses transfer quantum information back and forth between the resonator and the bismuth atoms, stored in the atoms spin states

Reference: James O’Sullivan, at el, : Random-Access Quantum Memory Using Chirped Pulse Phase Encoding, London Centre for Nanotechnology, UCL
Jarryd Pla, Chirping toward a Quantum RAM, APS Physics, November 7, 2022, Physics 15, 168

Quantum RAM (2)

- Quantum computers' central processor unit uses circuits made from superconductor metals wit Josephson Junction
- Quantum memory system's central processing is done with superconducting qubits– sending and receiving information via microwave photons
- At this moment, no quantum memory device can reliably producing quantum memory which can store these photons for long time.

Quantum RAM (3)

- Atomic spins store information time is order magnitude longer than superconducting qubits.
- Bismuth atoms embedded in silicon chips, the quantum information can store longer than second, the store time is much longer that superconducting qubits
- Atom spins requires complex Control and Measurement
- Hybrid approach is using superconducting qubits for process and atom spins for storage. How to transfer information between two different systems using microwave photons

Superconducting Qubits (100 qubits)	100 qubits per square millimeter	Semiconductor transistors (100 million transistors)	100 million transistor per square millimeter

Quantum Memory and Repeater

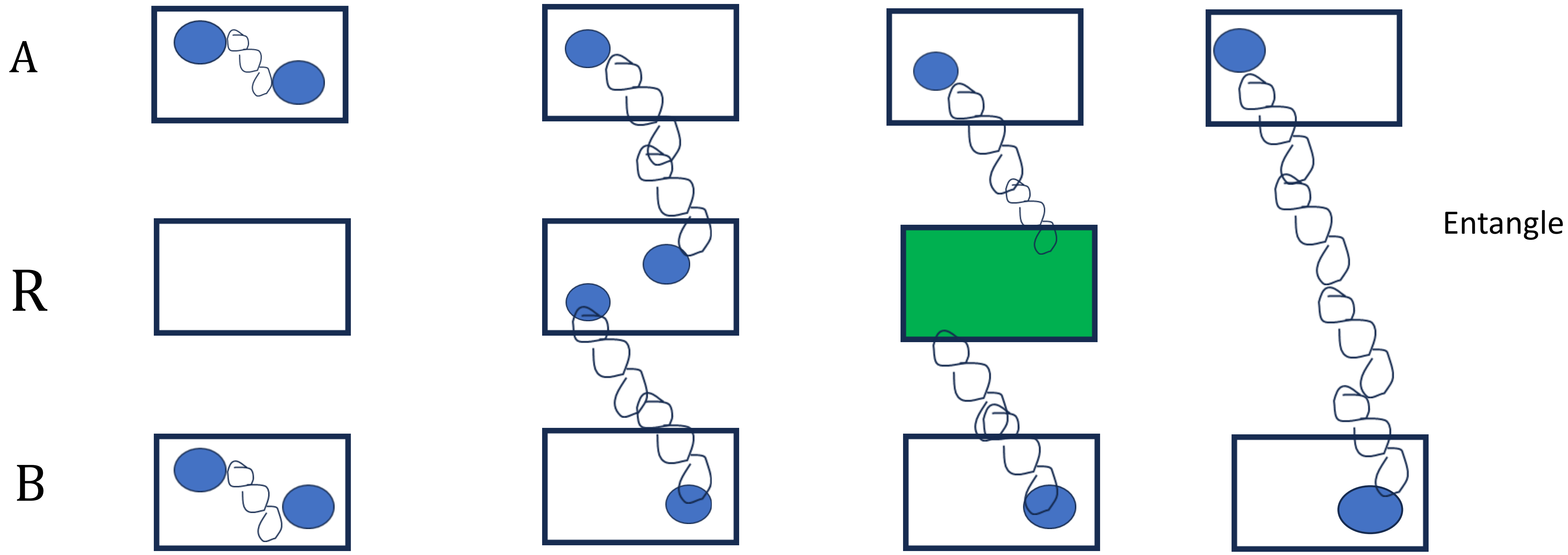
- Quantum repeater is a key element of quantum networks
- Long distance transmission requires quantum repeaters due to photon loss (fiber optic cables)
- In classical networks, the cable (fiber link) loss, we can use (repeater) multiple amplifiers to boost (amplify) the weak signals by producing many copies of he input photons. But, quantum networks due to “no-cloning theorem”, an amplifier cannot copy a quantum state,
- Quantum repeater can extend the range of the quantum network (long distance fiber cable).

Quantum Repeater Architecture

- Quantum entanglement
- Quantum channel and loss, channel (link) is fiber optic cable
- No-cloning theorem
- Architecture
  - Quantum Nodes
  - Quantum Memory
  - Entangle SWAP gates
  - Bell measurement
- Entanglement distribution

Quantum Repeater Conceptual Design

(No-Cloning → cannot copy, cannot use amplifier)

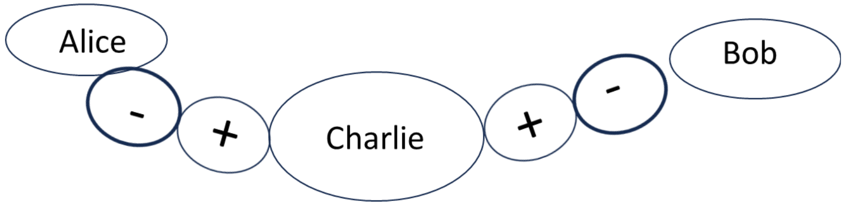


Source: QuTech Academy

Entanglement Swapping

Note: $\Phi^+ = \Phi_+$
$\Phi^- = \Phi_-$
$\Psi^+ = \Psi_+$
$\Psi^- = \Psi_-$

- Entanglement swap can be thought of as an extension of quantum teleportation
- Alice and Bob each share a two-qubits maximally entangle state with Charlie, C: $|\Phi^+\rangle_{AC_1}$ & $|\Phi^+\rangle_{C_2B}$

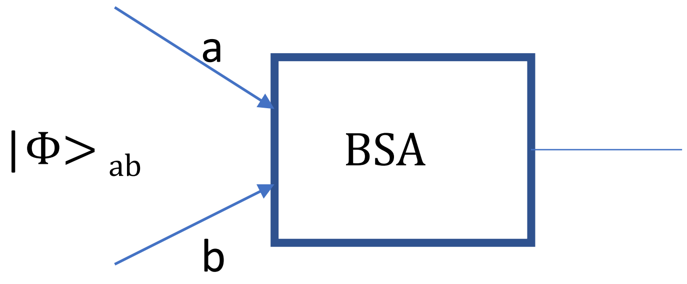


- $|\Phi^+\rangle_{C1C2} \rightarrow |\Phi^+\rangle_{AB}$ $\quad$ $|\Psi^+\rangle_{C1C2} \rightarrow |\Psi^+\rangle_{AB} = X_B|\Phi^+\rangle_{AB}$
- $|\Phi^-\rangle_{C1C2} \rightarrow |\Phi^-\rangle_{AB} = Z_B|\Phi^+\rangle_{AB}$ $\quad$ $|\Psi^-\rangle_{C1C2} \rightarrow |\Psi^-\rangle_{AB} = Z_BX_B|\Phi^+\rangle_{AB}$
- Alice and Bob's qubits end up in one of the four Bell states, depending the measurement of the outcome

Entanglement Swapping (2)

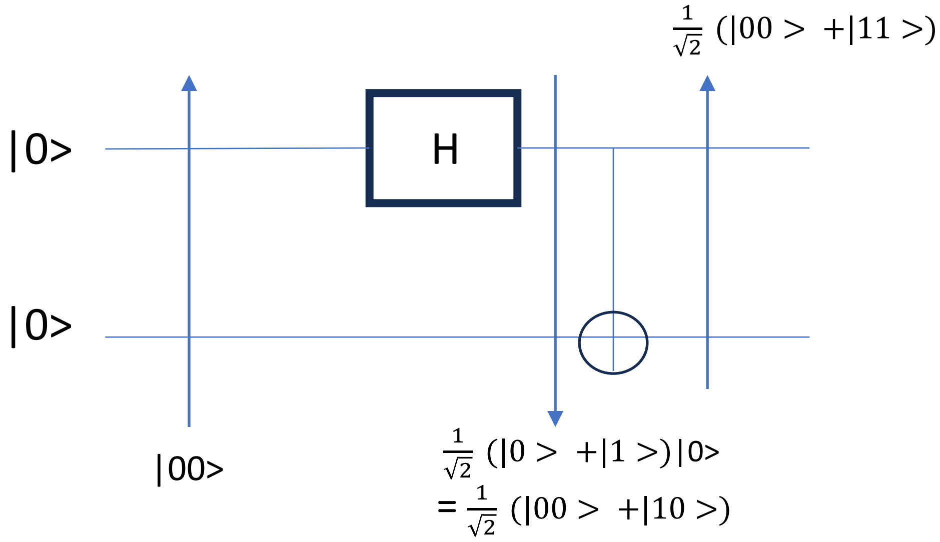
- Alice and Bob's qubits have not directly interacted, Alice and Bob established a entanglement state
  - Useful for Quantum Compunction
  - Entanglement can be propagated through a quantum network between two stationary nodes
  - Entanglement Swap is the a puzzling or difficult problem of quantum repeater schemes

Conceptual Bell State Analyzer



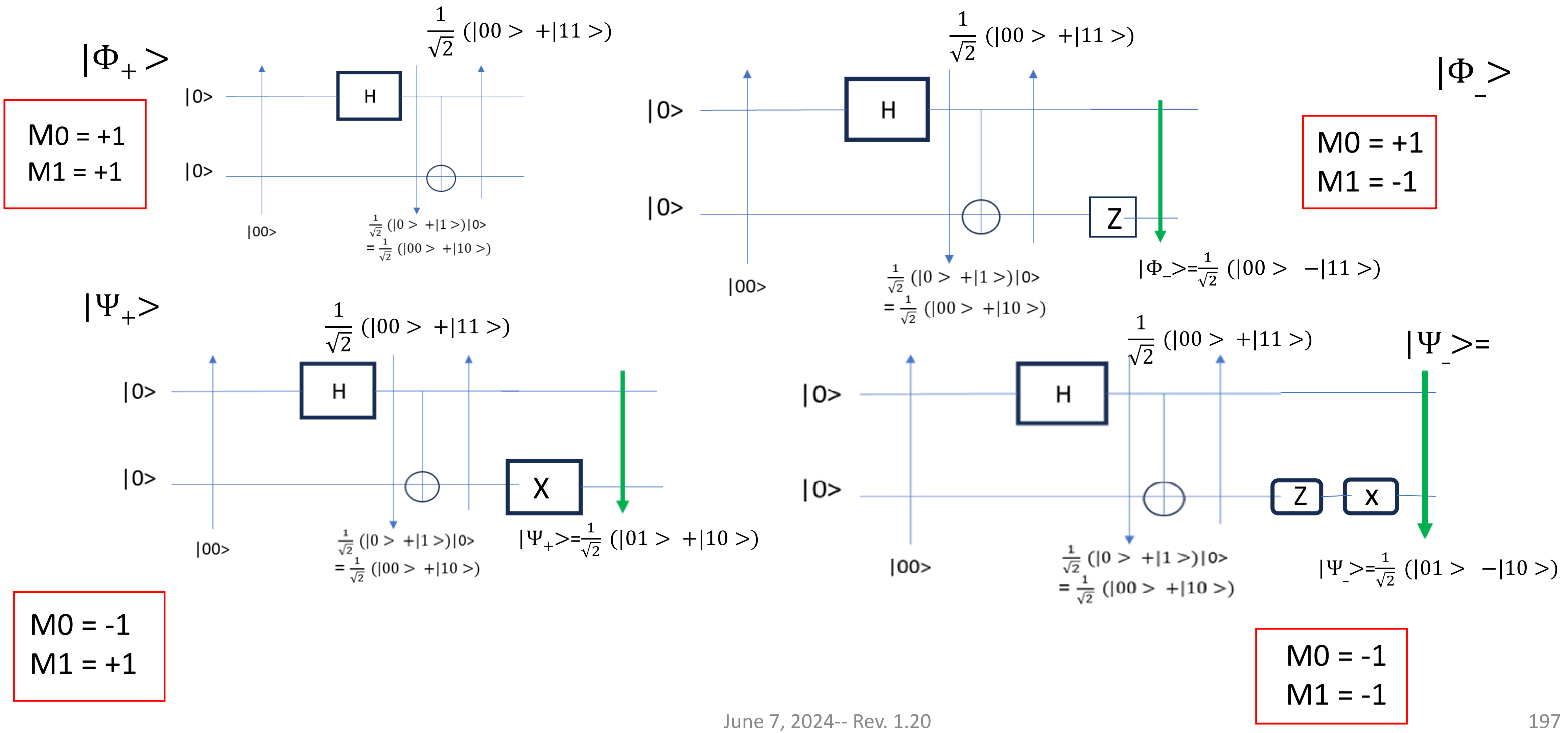
Bell States:

$$|\Phi_+ > = \frac{1}{\sqrt{2}} (|00> \ +|11>)$$
$$|\Phi_- > = \frac{1}{\sqrt{2}} (|00> \ -|11>)$$
$$|\Psi_+ > = \frac{1}{\sqrt{2}} (|01> \ +|10>)$$
$$|\Psi_- > = \frac{1}{\sqrt{2}} (|01> \ -|10>)$$

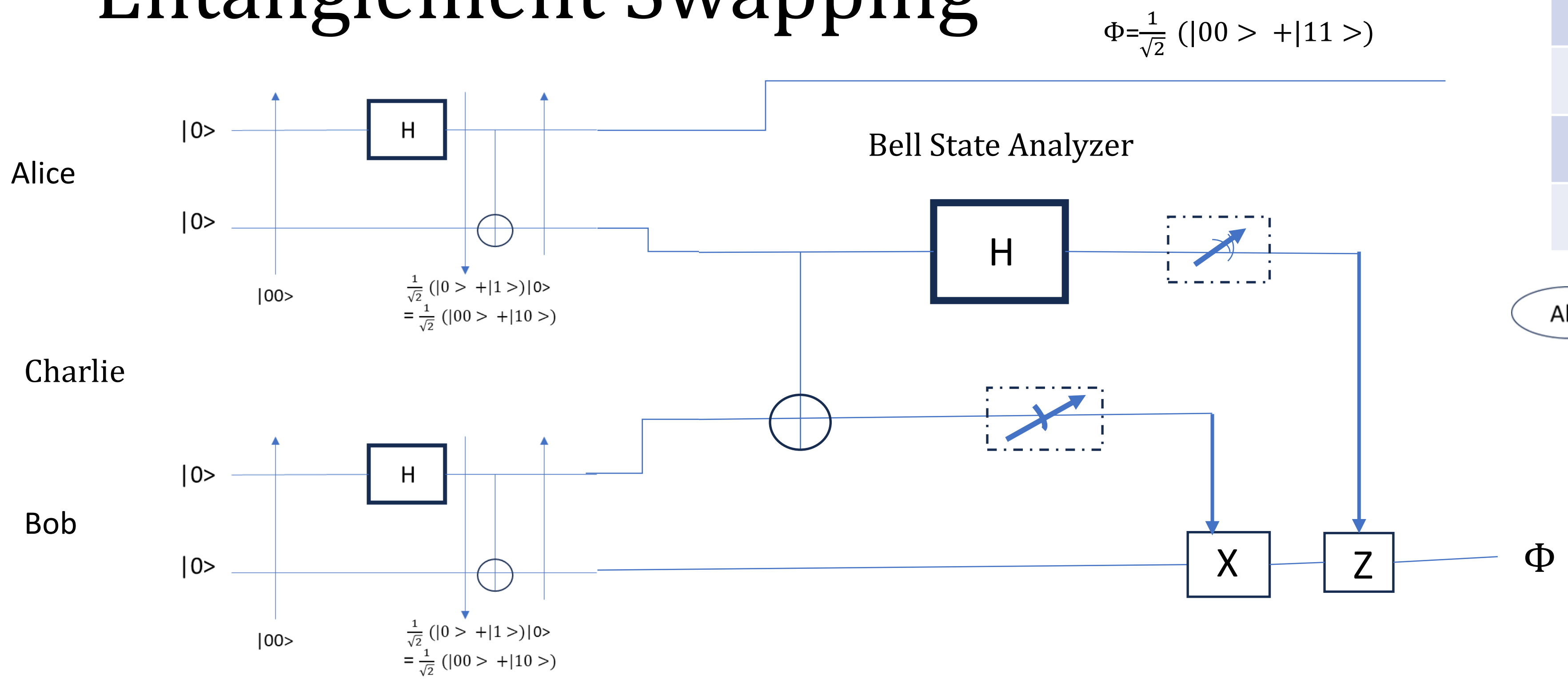


(Entanglement)

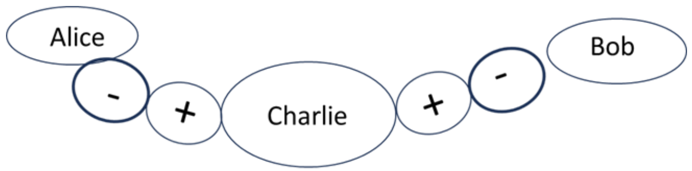
Preparing Bell States-A



Entanglement Swapping



m1	m0	Z	X
+1	+1	0	0
+1	-1	0	1
-1	+1	1	0
-1	-1	1	1



$|\Phi^+>_{C1C2} \rightarrow |\Phi^+>_{AB}$

$|\Phi^->_{C1C2} \rightarrow |\Phi^->_{AB} = Z_B|\Phi^+>_{AB}$

$|\Psi^+>_{C1C2} \rightarrow |\Psi^+>_{AB} = X_B|\Phi^+>_{AB}$

$|\Psi^->_{C1C2} \rightarrow |\Psi^->_{AB} = Z_B X_B|\Phi^+>_{AB}$

m1	m0	Z	X
+1	+1	0	0
+1	-1	0	1
-1	+1	1	0
-1	-1	1	1

Entanglement Swapping (2) – Math Notes

Preparing Bell States

$$\frac{1}{\sqrt{2}}(|0>|0> + |1>|1>)\ \frac{1}{\sqrt{2}}(|0>|0>+|1>|1>)$$

$$= \frac{1}{2}(\ |00>|0>|0> + |01>|01>\ |10>|10>+|11>|11>)$$

Box 1, BSA

$$|00>_{ab} = \frac{1}{\sqrt{2}}(|\Phi_+>_{ab} + |\Phi_->_{ab})$$

$$|01>_{ab} = \frac{1}{\sqrt{2}}(|\Psi_+>_{ab} + |\Psi_->_{ab})$$

$$|10>_{ab} = \frac{1}{\sqrt{2}}(|\Psi_+>_{ab} - |\Psi_->_{ab})$$

$$|11>_{ab} = \frac{1}{\sqrt{2}}(|\Phi_+>_{ab} - |\Phi_->_{ab})$$

Apply Box 1, BSA

$$= \frac{1}{\sqrt{2}}[\ |\Phi_+>|\Phi_+> + |\Phi_->|\Phi_->+$$
$$|\Psi_+>|\Psi_+>+|\Psi_->|\Psi_->]$$

Entanglement Swapping (3) – Math Notes

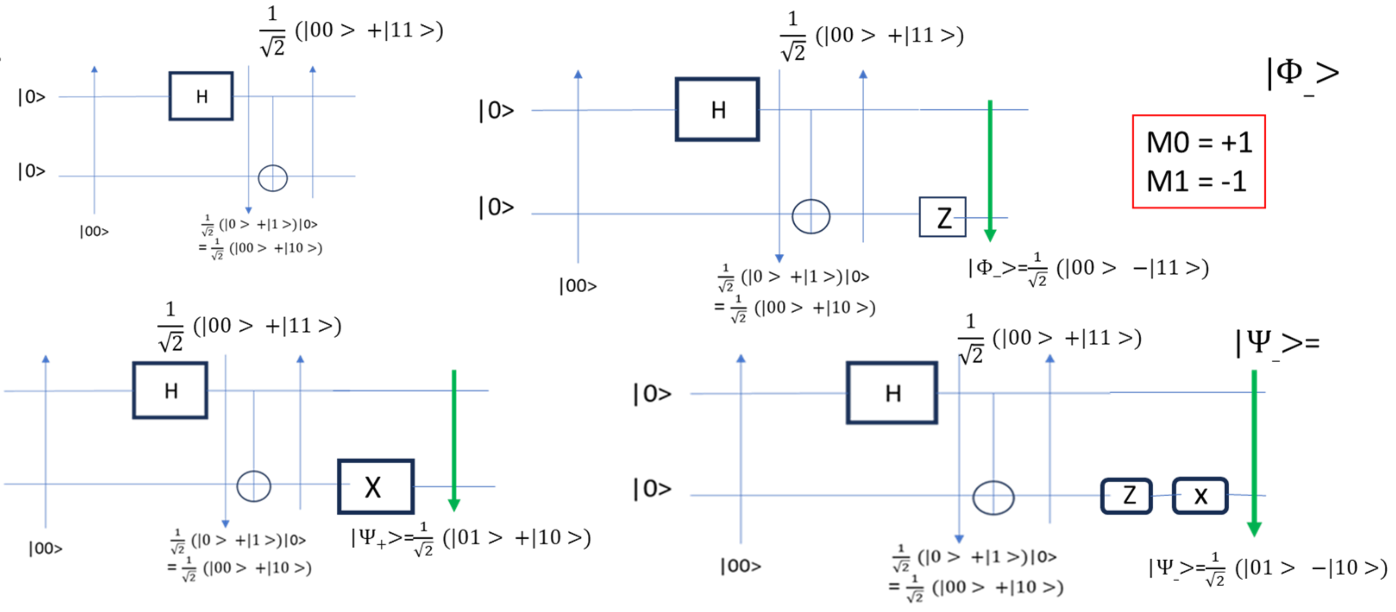
m1	m0	Z	X
+1	+1	0	0
+1	-1	0	1
-1	+1	1	0
-1	-1	1	1

Preparing Bell States-A

$|\Phi_+>$

M0 = +1
M1 = +1

$|\Psi_+>$



M0 = +1
M1 = -1

M0 = -1
M1 = +1

M0 = -1
M1 = -1

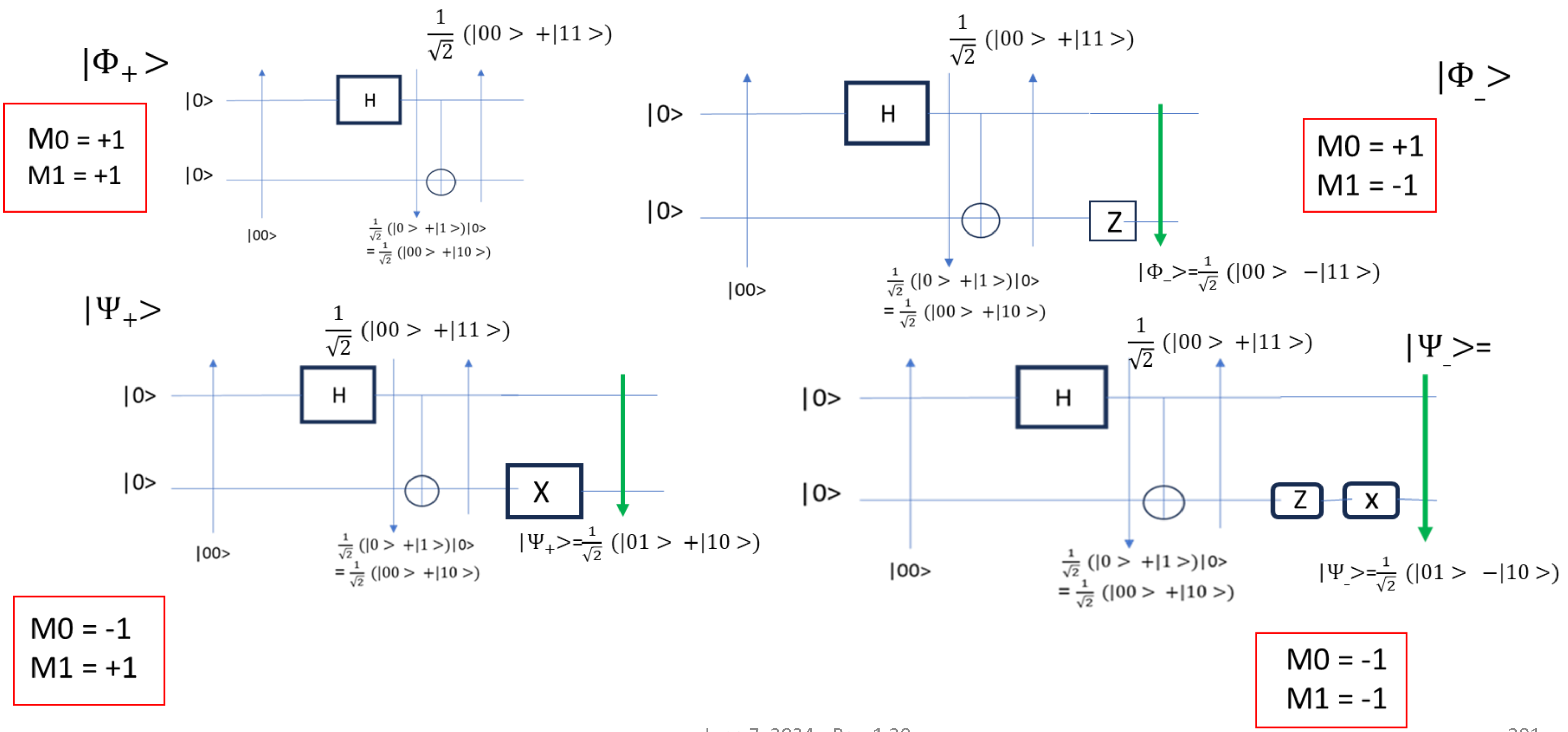
$$|\Phi_+> = \frac{1}{\sqrt{2}}(|00> +|11>)$$

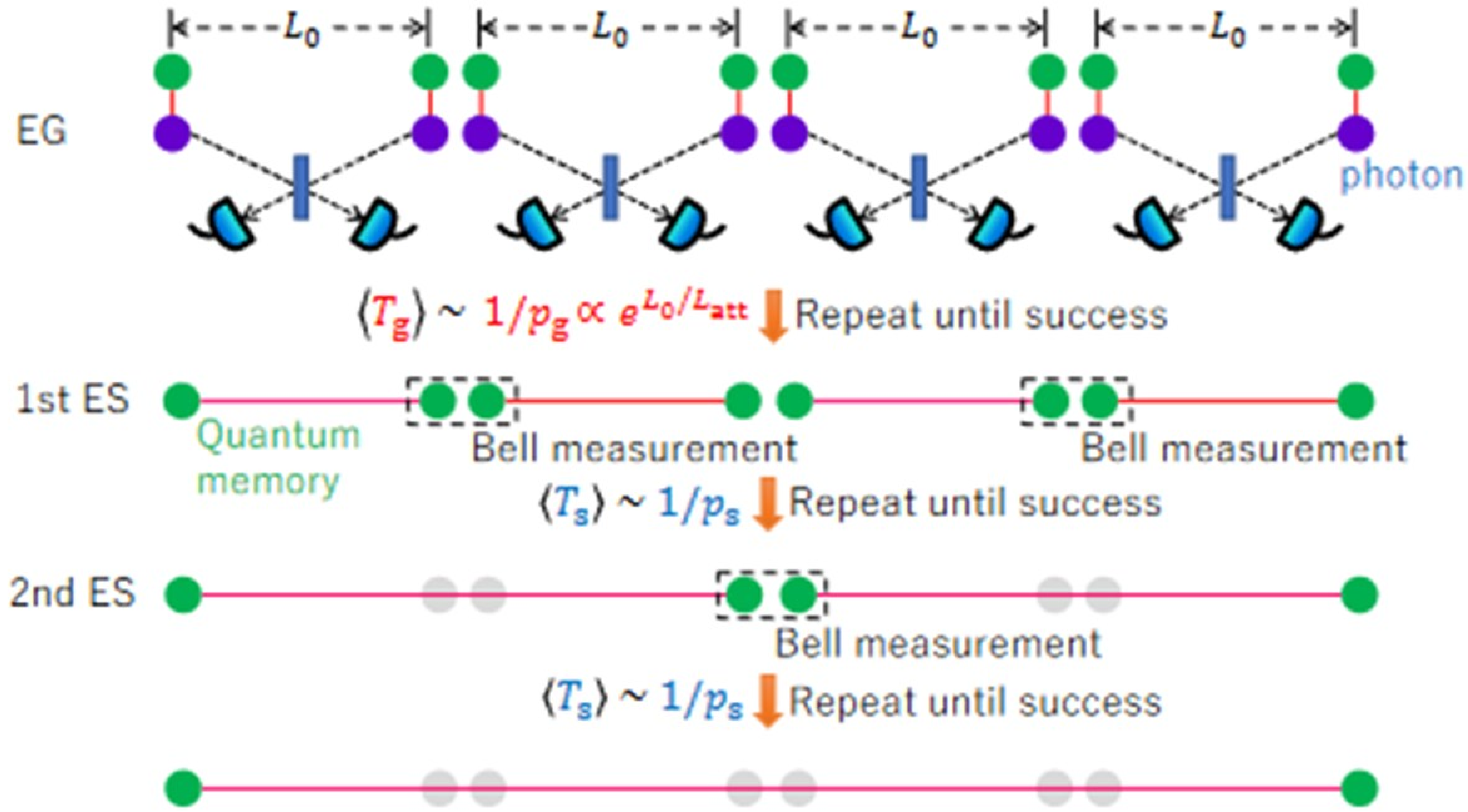
$$|\Phi_-> = \frac{1}{\sqrt{2}}(|00> -|11>)$$

$$|\Psi_+> = \frac{1}{\sqrt{2}}(|01> +|10>)$$

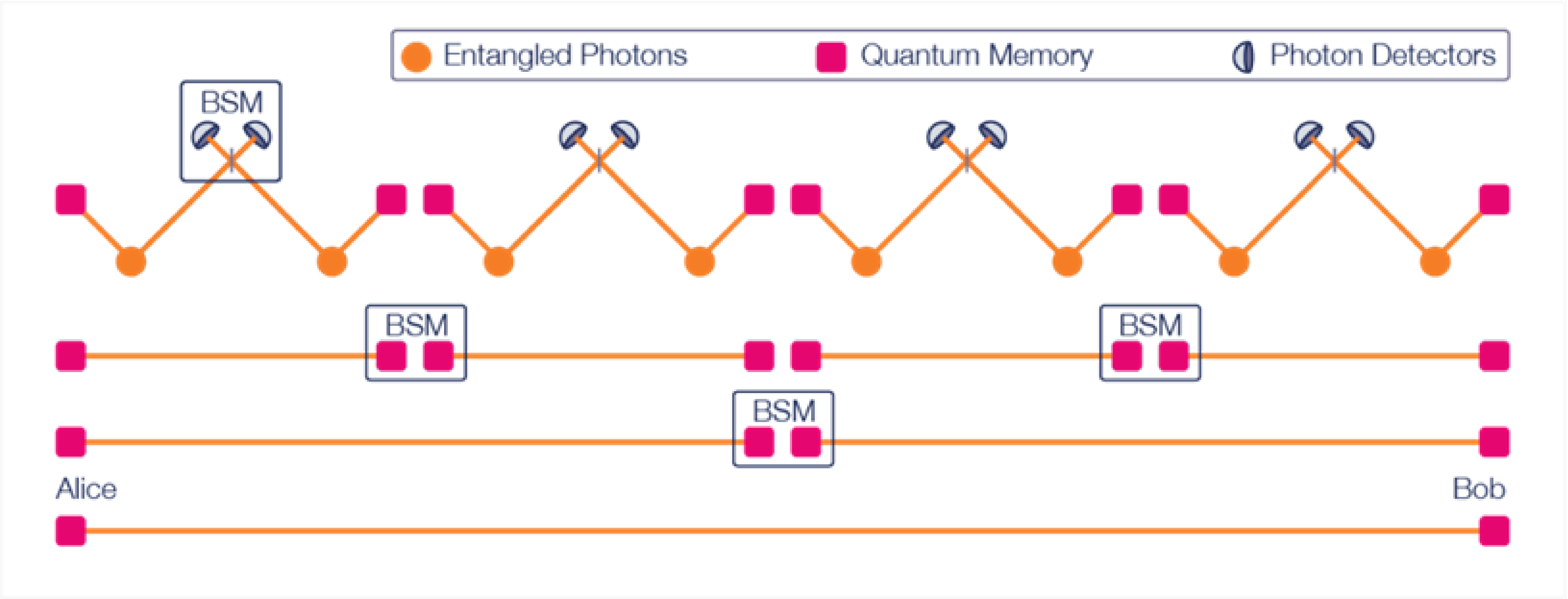
$$|\Psi_-> = \frac{1}{\sqrt{2}}(|01> -|10>)$$

Preparing Bell States-A





Reference: https://arxiv.org/pdf/2212.10820



Source: QuTech Academy

Summing up - Superconducting Qubits memory

- Superconducting circuit systems– most promising quantum computing
- Superconducting circuit systems – nonlinearity provided by Josephson junctions, quasi-atom structures provides quantum state manipulation
- Di Vincenzo Criteria
- Two level System, Bloch Sphere
- Qubit Gates
- Relaxation and Dephasing
- Superconducting qubits uses as computing registers

Quantum Computing and Semiconductor

A New Perspective

Cryogenic CMOS

CMOS Operates under very low temperatures-- 93^0K ($-180.15\ ^0C$)

Bulk CMOS

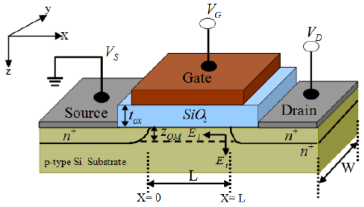
Major quantum mortalities use CMOS fabrication process

John Bardeen, William Shockley and Walter Brattain at Bell Labs in 1948. They invented the point-contact transistor in 1947 and bipolar junction transistor in 1948.

--Wikipedia

1958 J. Kilby/TI: Add metal connection, not able production in high Qty.

1959: R. Noyce, Intel, First true monolithic IC chip, IC Fabrication process.



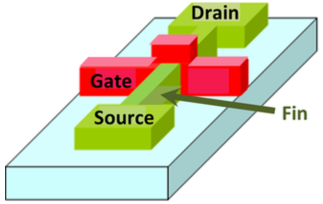
Basic MOSFET

Mohamed Atalla (left) and Dawon Kahng (right) invented the MOSFET (MOS transistor) at Bell Labs in 1959.

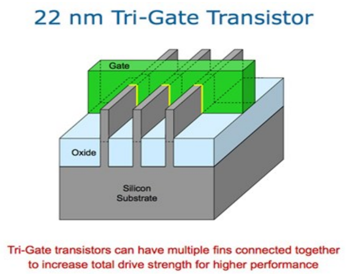
1963; Chih-Tang Sah and Frank Wanlass at Fairchild Semi. Invented CMOS

--Wikipedia

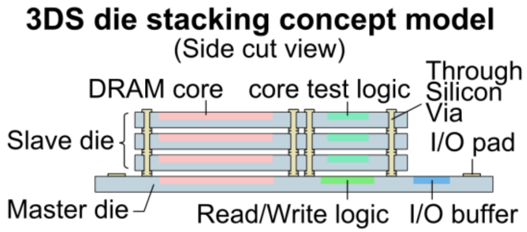
1974 Robert H. Dennard with IBM teams' paper: MOSFET scaling, Dennard Scaling.



FinFET (3D transistor)



2011: Intel's Tri-gate FinFET transistors(22nm)



3D Die stacking

History of MOSFET/CMOS

1947: J. Bardeen, W. Brattain, and W. Shockley– Transistor (Bell Lab.): Replaced the vacuum Tube.

1958: J. Kilby – Add metal connection as a layer on the top of IC chip(Silicon).

1959: R. Noyce -First IC process; Noyce's fabrication using plasma process, Kilby is metal wire, Not able production in high volume.

1959: DaWon Kahng/Mohmed M. Atalla (Bell Lab): Invented modern MOSFET

1963: Chih-Tang Sah/Frank Wanlas (Fairchild Semi., Mountain View, CA) invented CMOS.

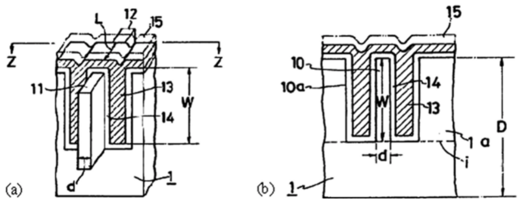
1971: Intel released 4004, first CPU

1974: Dennard with his IBM team published the papers: MOSFET scaling, Dennard scaling

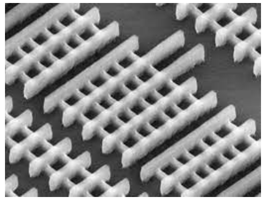
1989: FinFET: The first FinFET transistor type was called a "Depleted Lean-channel Transistor" or "DELTA" transistor, which was first fabricated in Japan by Hitachi Central Research Laboratory 's Digh Hisamoto, Toru Kaga, Yoshifumi Kawamoto and Eiji Takeda in 1989.

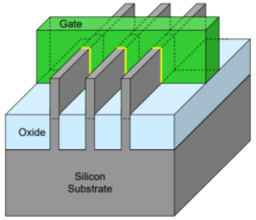
1997: (DARPA), awarded a contract to University of California, Berkeley to develop a deep sub-micron transistor based on DELTA technology led by Hisamoto along with Chenming Hu

2011: Intel released "Big change" 22nm FinFET based processor (Ivy Bridge), full production, 2012.



Source: Hitachi





Source: Intel

Challenges– CMOS Technology

- Logic and Memory silicon's performance is solely based on CMOS scaling down.– followed Moore's law
- Transistor slows down when CMOS transistor reached 10nm and below.
- High manufacturing cost, limited applications can use the technology below 10nm.
  - impacting on new(invented) ideas, due to high cost and limited wafer suppliers
- New computer architectures without rely on transistor scaling down.

Economy and Technology

Economy:

- Conventional Logic Array FPGA’s and Logic Silicon’s performance improvements solely relied on technology scaling down.
- Technologies below 10nm are facing multiple challenges:
  - High static power consumption
  - Complex manufacturing process leading to,
    - reduced performance gain, reduced reliability, complex testing process, extremely costly masks, and low yield.
- High manufacturing cost– requires a GDP size Fab
- Only suitable for limited applications

Technology:

- AI, Machine learning Computation-in-Memory design architectures , the preferred technology is Non-Volatile Memory elements.
- Non-volatile Memory technology is about one or two generation behind traditional CMOS process.
  - Logic (Gate) Array FPGA requires scale-down process to achieve desired performance. – 7nm, 5nm,etc.
  - Non-volatile memory technology may use 3D Layers stacking on the same die to achieve high density
  - The industry has not able to demonstrate means to stack memory on top of logic gates on the same die.

Technology (2):

- Logic (CPU and GPU), based on CMOS process, continues to consume large die area, high power without performance improvements
- CMOS process scales below 28nm, the technology can no longer provide excellent reliability, and cost-effective solutions for memory (DRAM) and logic silicon.
- DRAM --- RowHammer, a by-product of technology scaling
- NAND Flash technology is back to use 45nm for achieving realizable NAND cells with complex ECC to correct errors.

Quantum Computing needs Semiconductor Technology

All Electronics need Semiconductor Chips including Quantum Processor Unit, Communication Unit, and Quantum State Controllers.

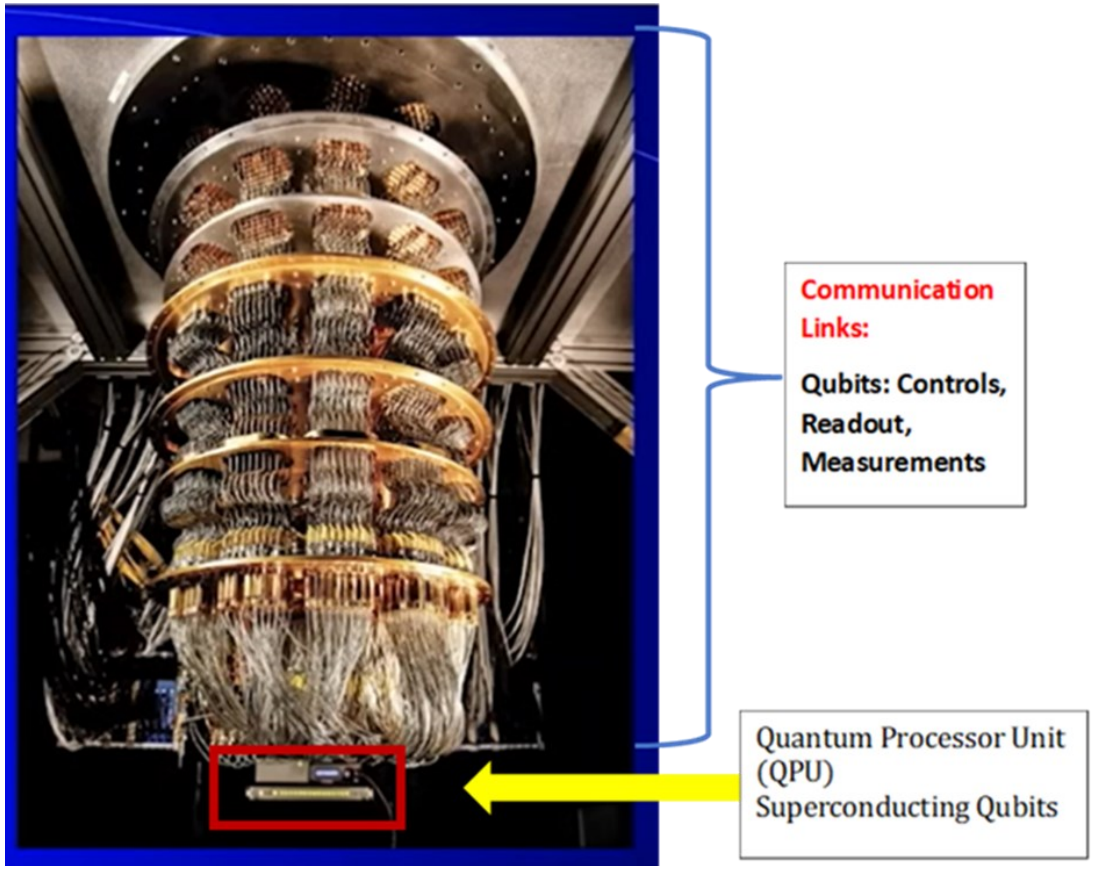
Appendix 4:

a. Notes of Quantum Computer Hardware design—Dec 30, 2023

b. How to Scale Up? – Dec 22, 2023

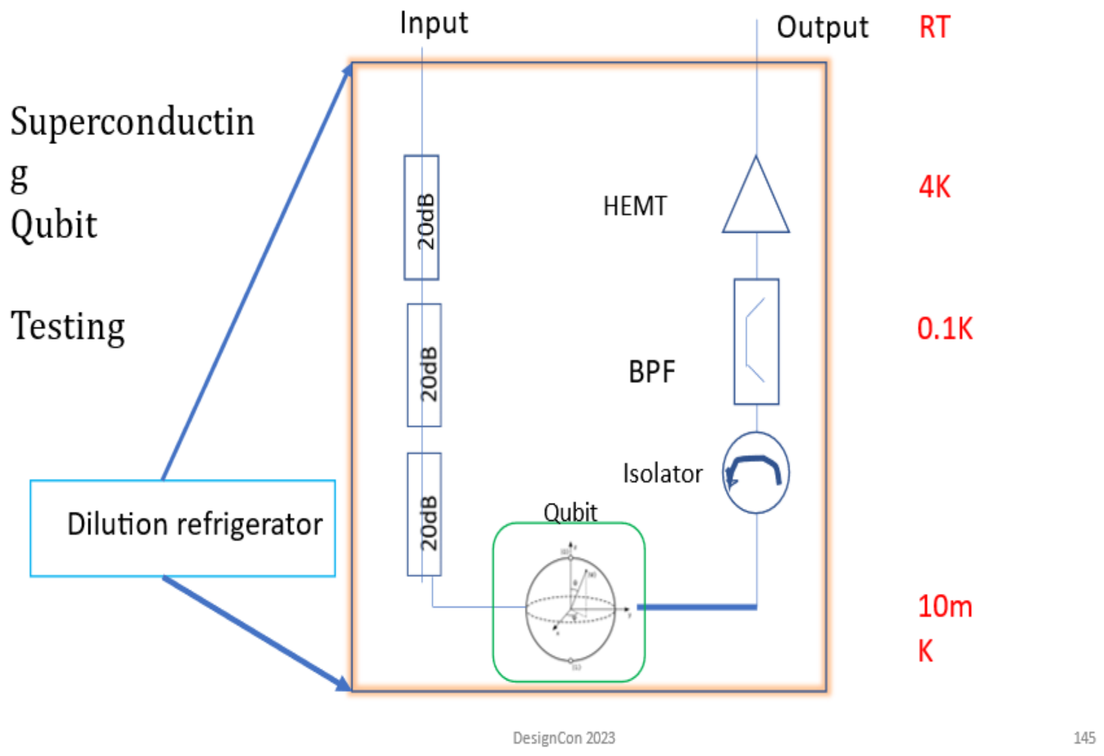
Quantum Computer Hardware structures –
three function blocks

- Quantum Processor Unit (QPU) consists Qubits silicon with other silicon elements
- Communication Links: Qubits Controls, Readout, Measurements etc. The links operated under low temperature to room temperature.
- External (room temperature) control units and computers, etc.

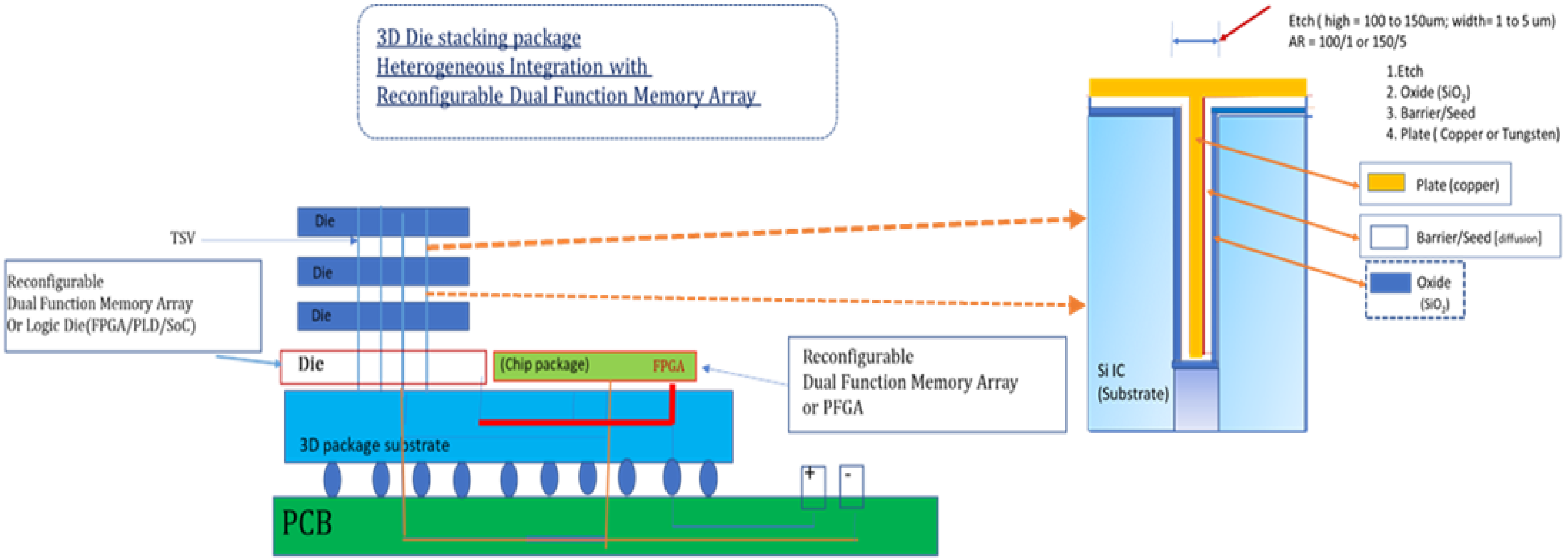


Building Quantum Computer hardware --challenges

- Low Temperature (milli Kelvin Temperature)
- Integration of control and readout that maintain Qubit coherence at low temperature
- 3D integration technology is required

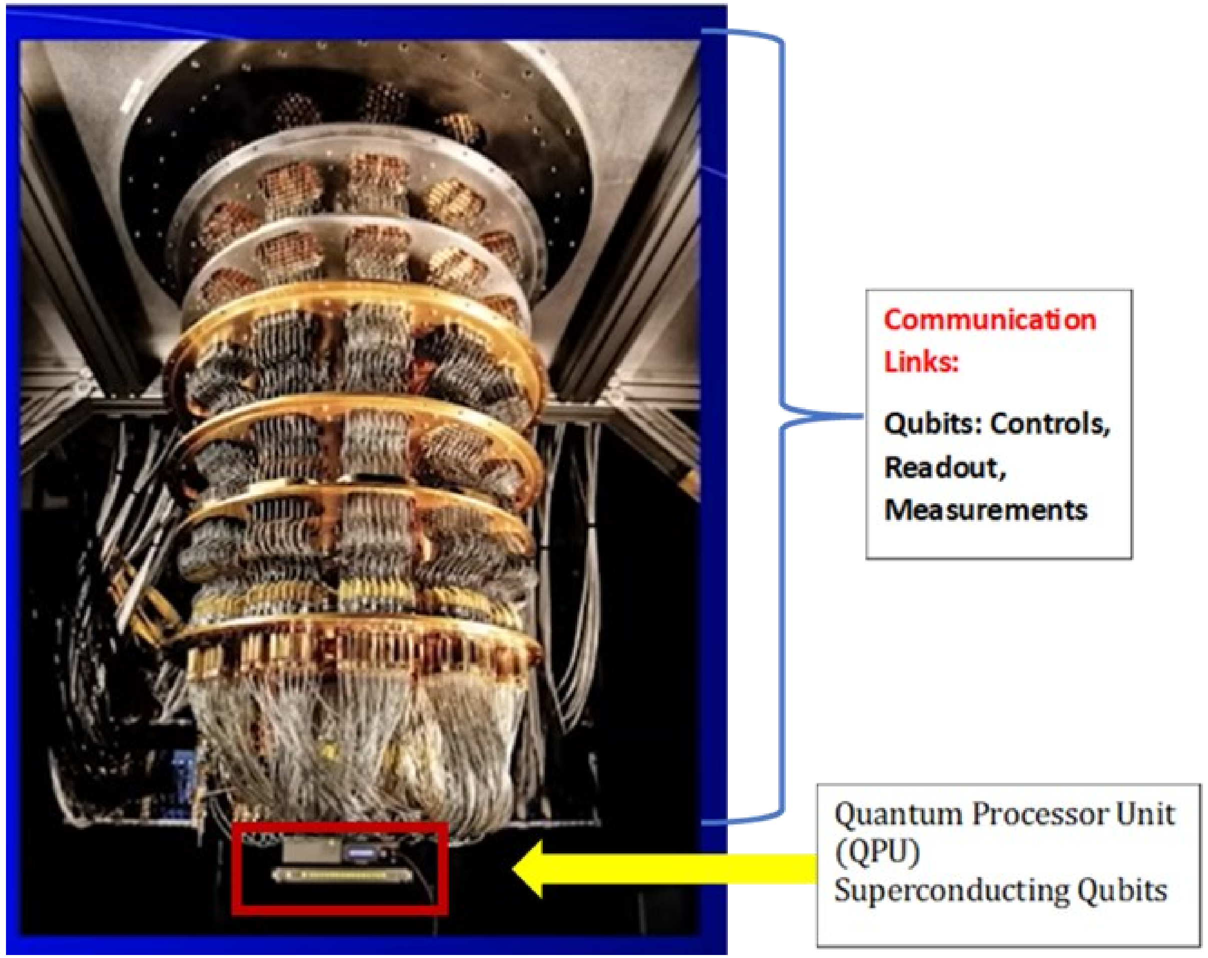


TSV and Die Stacking



How to scale up Quantum Computer

- To achieve large scales of Quantum Computers requires completely new and stable qubits
- Requiring solid or advanced communication to connect the external (room temperature) control and read-out equipment to low-temperature qubits (Superconductor Qubits).
- The electronic circuits have to be very accurate; many circuits have to be placed to close the low temperatures' qubits; the qubits require many analog and digital circuits to control Qubits



Quantum Computer Hardware Design and Semiconductor Technology

--Quantum Computer company must operate like a Fabless semiconductor company.

-- All the components are conventional CMOS technology. We are designing a CMOS-based computer system, and semiconductors are key elements.

Large-Scale Quantum Computers

- Qubits require a large number of analog and digital circuits to control Qubits.
  - Large numbers of interconnect/entangled cables of electronics circuits to control and measure Qubits create a bottleneck for Quantum Computer to scale to Large Quantum Computers.
- Qubits must operate at low temperature, $T = 10mK$.
- Cryo-CMOS
  - Control and measurement circuits operating under low temperatures.
  - Low power

Low temperature impact on Bulk CMOS electric characteristics

- The bulk CMOS-based production products have trouble yielding a reasonable number of devices for temperatures reaching -65^{0}C.
- Noise conditions increased with exponential curves.
- Many yield devices have to relax the product specifications.
- Mobility μ increased at low temperature
- Threshold Vt increased at low temperature (Vt increases 40% from RT to 20mK)

Cryogenic CMOS– Challenges

- The classical bulk CMOS operates under extremely low temperatures, and the circuit functionality is hard to predict.
- The transistor's threshold voltage could increase to 40%, and the circuits cannot operate as expected or simulated results.
- Circuits Designers cannot control the circuit's accuracy.
- It may force designers and devices physicist to produce new Cryogenic CMOS with entirely different technologies and materials.
- Cryogenic CMOS technology has to be well defined and fully characterized on circuits.

Cryogenic CMOS– Challenges (2)

- Cryogenic CMOS technology has to be well defined and fully characterized on circuits. Otherwise, we may create more unknown issues (unstable conditions) to control and measurement of Qubits.

- Cryogenic CMOS requires extensive research and development efforts. It may require significant investments of time and resources to achieve the objectives.

Cryogenic CMOS (3)

- Many research works and excellent papers were published that demonstrate the potential to use Cryogenic CMOS circuits placed inside the dilution refrigerator.
- There are outstanding arguments to put the readout circuit/control logic inside the dilution refrigerator—the development efforts to develop new Cryogenic CMOS circuits and PCBs wires under different temperatures. The last task (PCB wires) is much simpler.

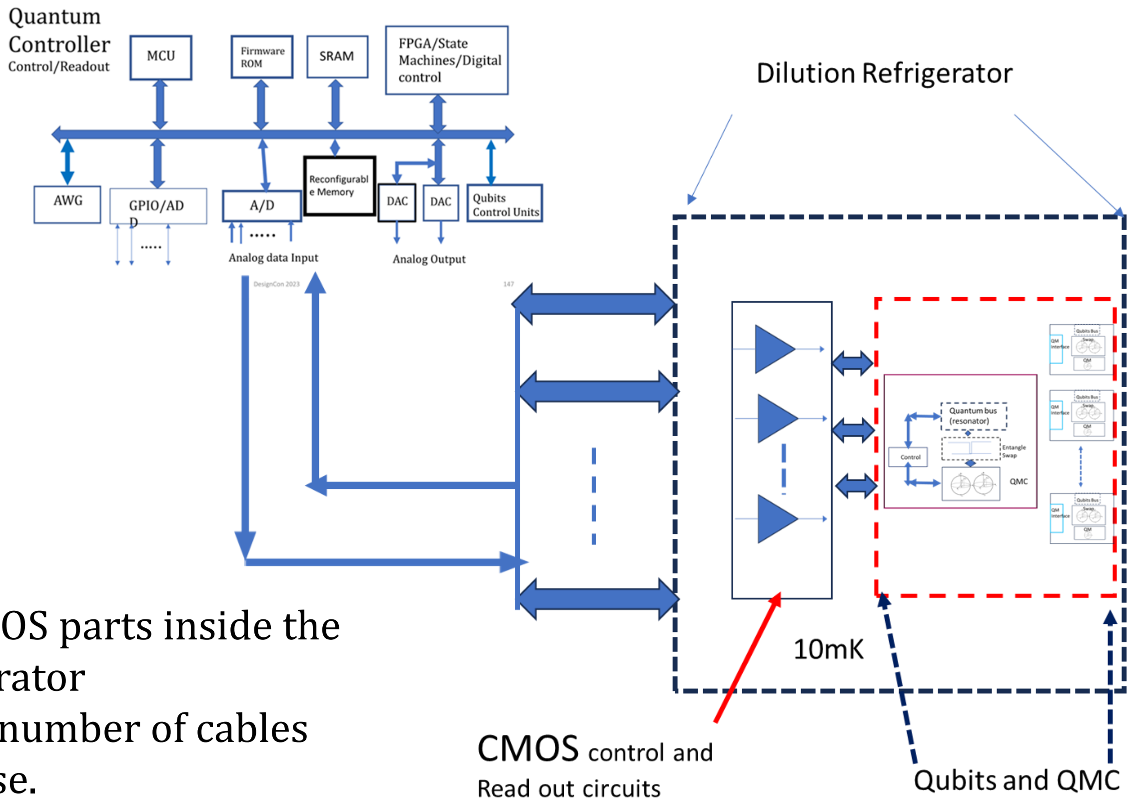
Quantum Computer QPU control/read-out circuits design: Cryo-CMOS (examples)

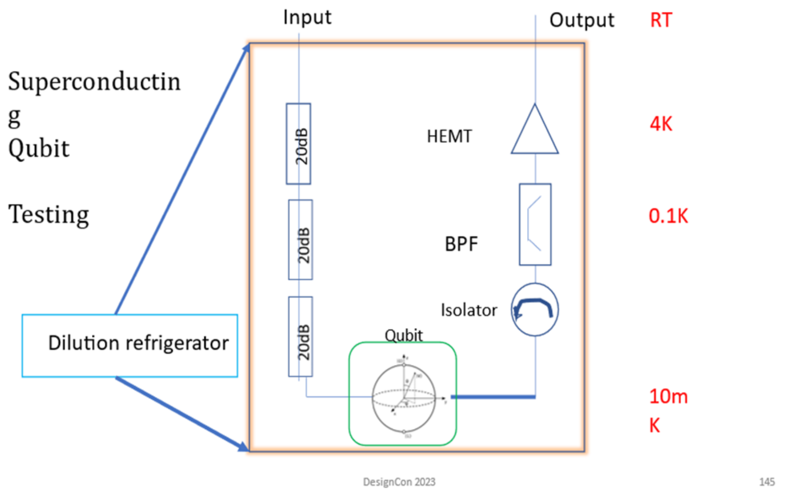
- DAC
- Superconducting Qubit –Resonator systems
- HEMT
- Qubit Basic Measurement Setup

Design parameters under low temperatures--
impacts on Analog Circuits

- Transistor (CMOS) electric characterizes under low temperature
- Temperature dependence—Threshold, Mobility, Flicker noise (1/f)
  - Threshold voltages, $\frac{dV_t}{dT}$ = -2.1mV/^{0}C @ Vdd= 2.8V (T= -55 ^{0}C to 125 ^{0}C)
  - Threshold voltages, $\frac{dV_t}{dT}$ = -?.??mV/^{0}C @Vdd = 1.2V (T=-273.14 to RT)
  - Mobility, (1/μ) $\frac{d\mu}{dT}$ ~ - ?K ppm/^{0}C
- Second order effect
- Difficulty to design Analog circuits with digital technologies
- Temperature coefficients are the challenges technical issues for Analog circuit design

Put all the parts together- Quantum Computer.





Place all the CMOS parts inside the Dilution refrigerator

1. Reduce the number of cables
2. Control noise.
3. Reduce race conditions between cables/signals
4. Clock skew.

Re-sources:

Quantum Software for developers

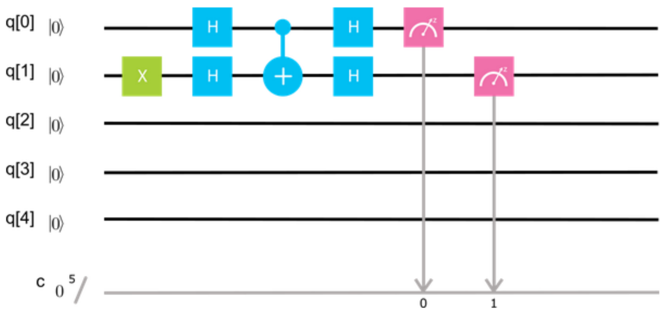
Re-sources:
Quantum Software for developers

- IBM Quantum Experience and Open-Source Quantum Development
  - https://quantum-computing.ibm.com/
  - https://qiskit.org/
  - IBM Quantum Composer

Q1IBMQE5: N=1 Data Qubit, Balanced Function

1.0/1.0 point (graded)

In the console below write the QASM code that generates the following circuit:



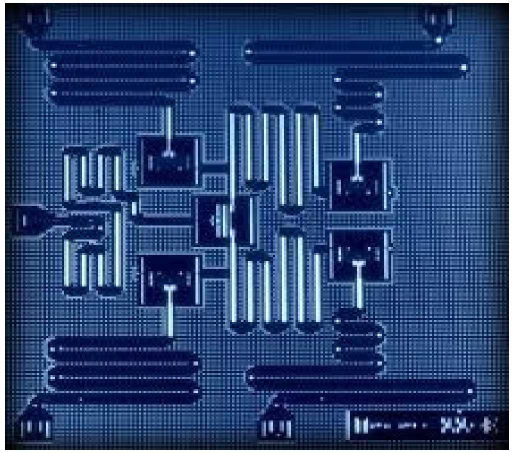
Submit your response, which will be evaluated with the grader, employing a numerically simulated quantum computer. Once you have a correct submission, your QASM code will automatically be queued to run on a real quantum computer at IBM.

```
include "qelib1.inc";

qreg q[5];
creg c[5];

x q[1];
h q[1];
h q[0];
cx q[0],q[1];
h q[0];
h q[1];

measure q[0] -> c[0];
measure q[1] -> c[1];
```

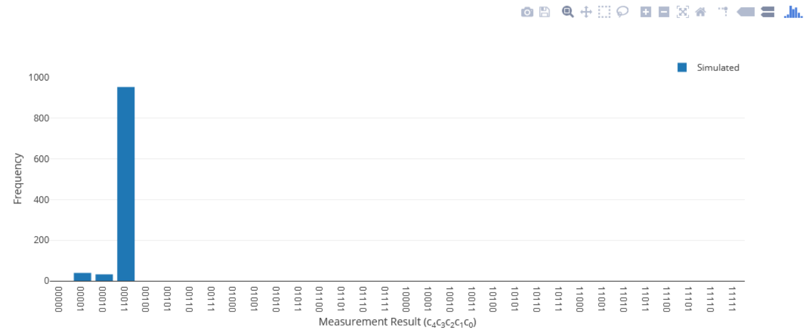
Press ESC then TAB or click outside of the code editor to exit

Correct

Source: IBM 5 Qubits



Correct



Show answer

IBM QE: QASM Simple Quantum Algorithm in Practice (simulation)

Source: https://quantum-computing.ibm.com/

Quantum Software and Classical (Standard) Software

Classical (Standard) Software	Quantum Software	
Schematic Capture– Circuit Input/CAD tools -- Circuit simulation- circuit timing, power	Quantum Circuit Composer	
Complier– VHDL	Quantum Complier, Ex. IBM QASM	
Chip Layout- Place & Route (Hardware/Technology Specific)	Hardware Specific Mapping Ex. Superconductors, Trapped Ions	
Circuit Simulation- circuit timing, power Ex. SPICE	Device Simulation– device simulation, noise/errors	
Silicon Testing– wafer sorting, verification/characterization, final testing	QVCC-quantum validation, verification/characterization of Qubits and Devices.	

Quantum Algorithm/Circuit Development Software

IBM Qiskit Company: IBM License: Apache-2.0 Open Source: Yes Website: https://qiskit.org/ Standard(host) Language: Python Quantum Language: QASM	**Cirq** Company: Google License: Apache-2.0 Open Source: Yes Website: https://quantumai.google/cirq Standard(host) Language: Python
Forest Company: Rigetti License: Apache-2.0 Open Source: Yes Website: https://qcs.rigetti.com/sdk-downloads Standard(host) Language: Python Quantum Language: Quil	**QDK** Company: Microsoft License: MIT Open Source: Yes Website: https://azure.microsoft.com/en-us/resources/development-kit/quantum-computing/#overview Standard(host) Language: Python Quantum Language: Q#

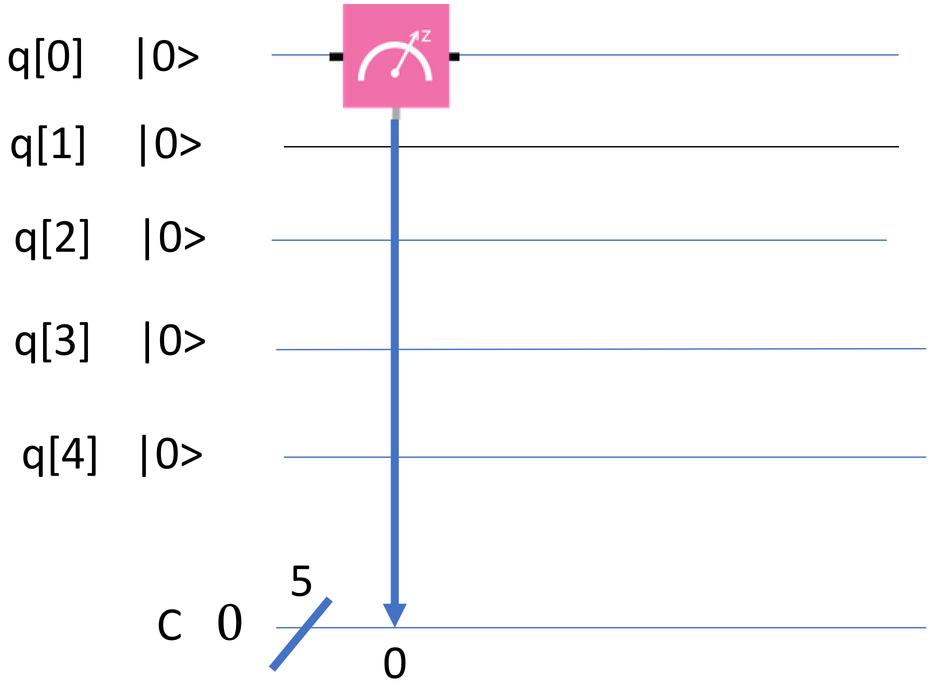
IBM QASM

IBM-QASM (Circuit Composer)

- Creating a Quantum circuit using the Circuit Composer

Example1: QASM codes create a quantum circuit

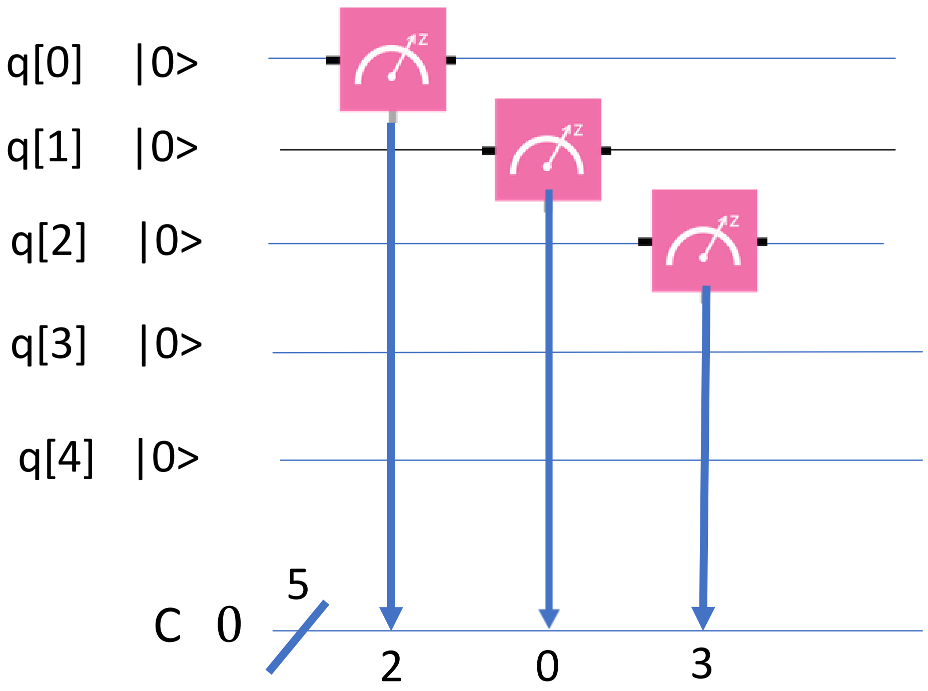
```
include "qelib1.inc";
qreg q[5];
creg c[5];

// This is a comment
measure q[0] -> c[0];
```



$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$

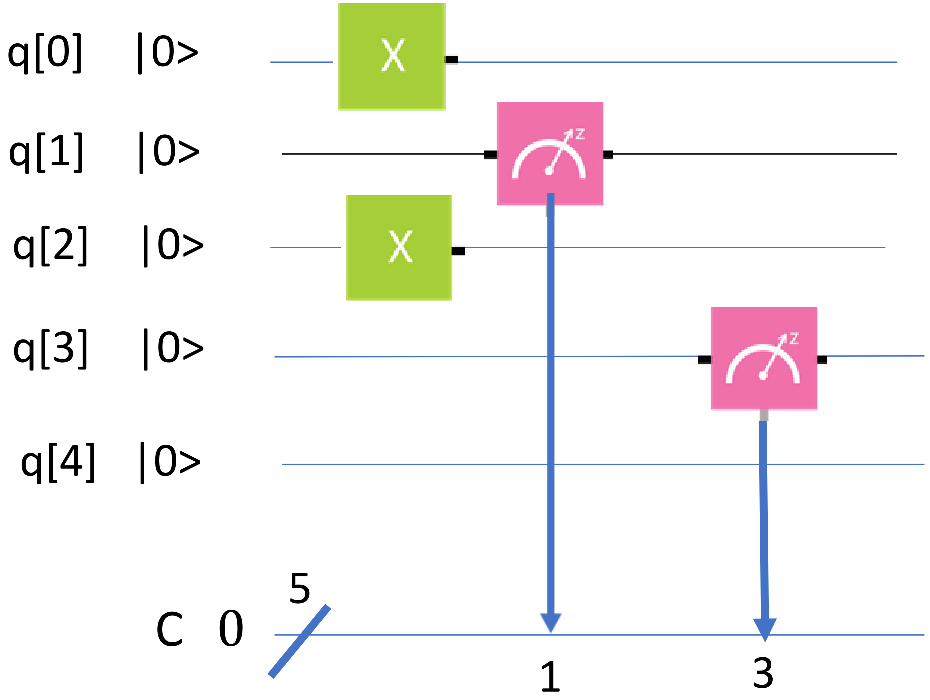
- Example2-Measure

```
1 include "qelib1.inc";
2 qreg q[5];
3 creg c[5];
4 measure q[0] -> c[2];
5 measure q[1] -> c[0];
6 measure q[2] -> c[3];
```

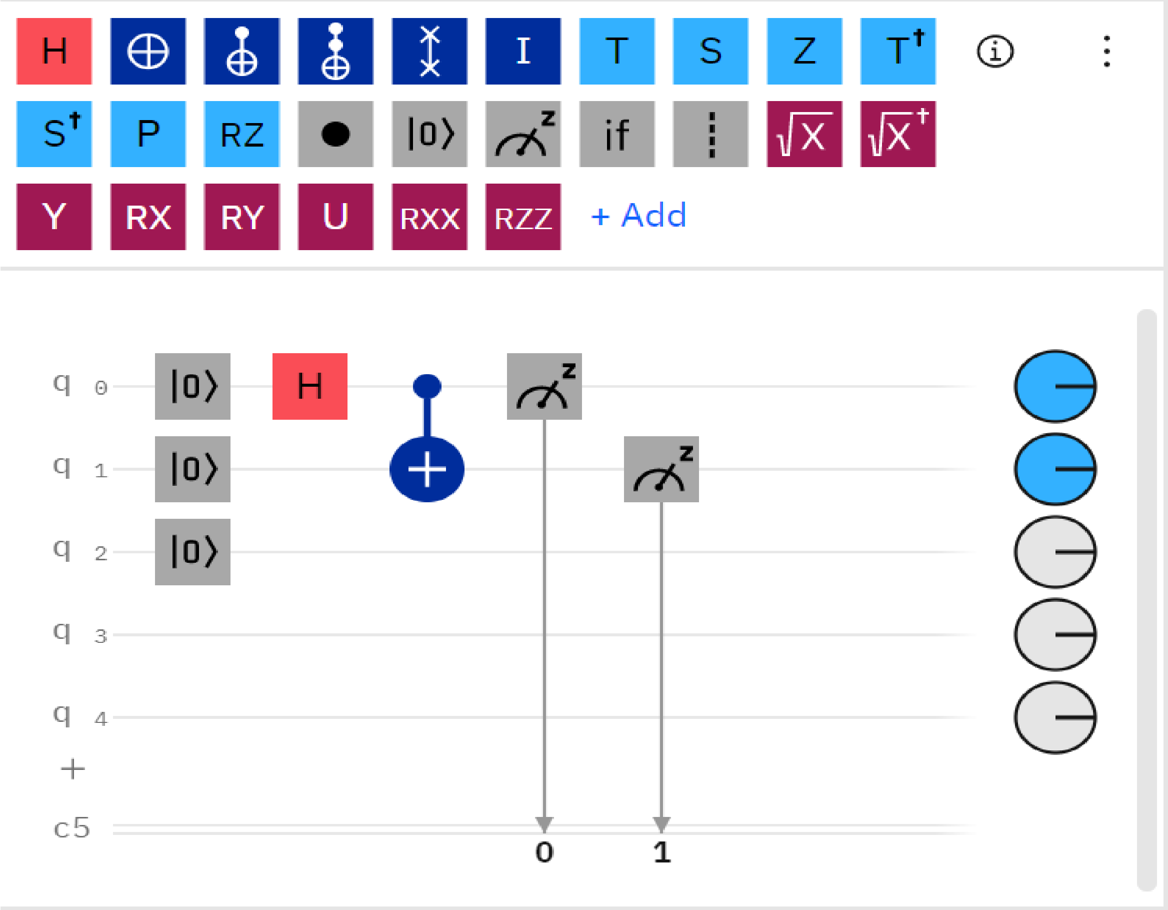


- Example3-Measure

```
include "qelib1.inc";
qreg q[5];
creg c[5];


x q[0];
x q[2];
measure q[1] -> c[1];
measure q[3] -> c[3];
```



- Example 4 Entanglement –Circuit



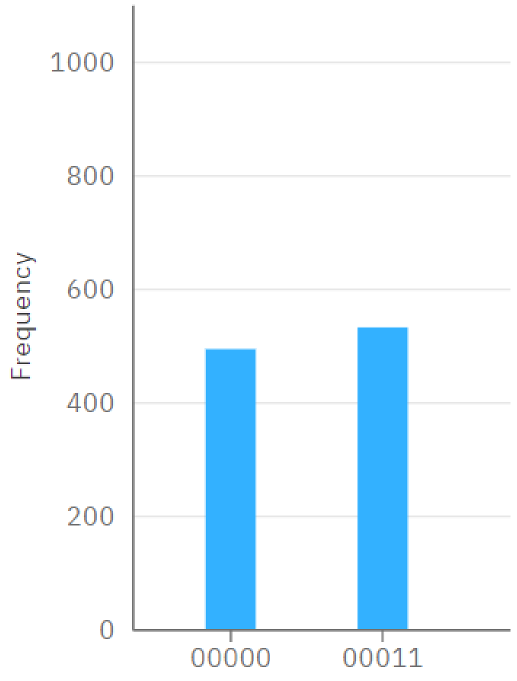
OpenQASM 2.0

Open in Quantum Lab

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[5];
creg c[5];

reset q[0];
reset q[1];
reset q[2];
h q[0];
cx q[0],q[1];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

• Example 4 Entanglement - results

Histogram



Circuit

Diagram | **Qasm** | Qiskit

Original circuit

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[5];
creg c[5];

reset q[0];
reset q[1];
reset q[2];
h q[0];
cx q[0],q[1];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

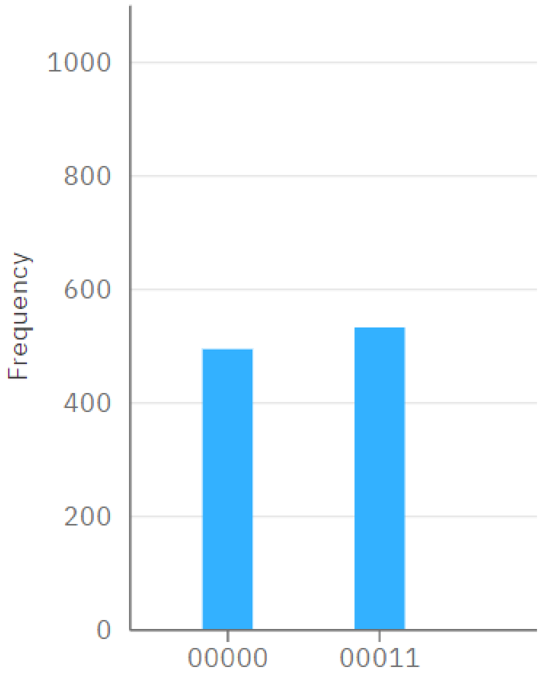
Transpiled circuit

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[5];

creg c[5];


h q[0];
cx q[0], q[1];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

Measurement outcome

• Example 4 results -- Qiskit

Histogram



Circuit

Diagram | Qasm | **Qiskit**

Original circuit

```
from qiskit import QuantumRegister,
ClassicalRegister, QuantumCircuit
from numpy import pi

qreg_q = QuantumRegister(5, 'q')
creg_c = ClassicalRegister(5, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.reset(qreg_q[0])
circuit.reset(qreg_q[1])
circuit.reset(qreg_q[2])
circuit.h(qreg_q[0])
circuit.cx(qreg_q[0], qreg_q[1])
circuit.measure(qreg_q[0], creg_c[0])
circuit.measure(qreg_q[1], creg_c[1])
```

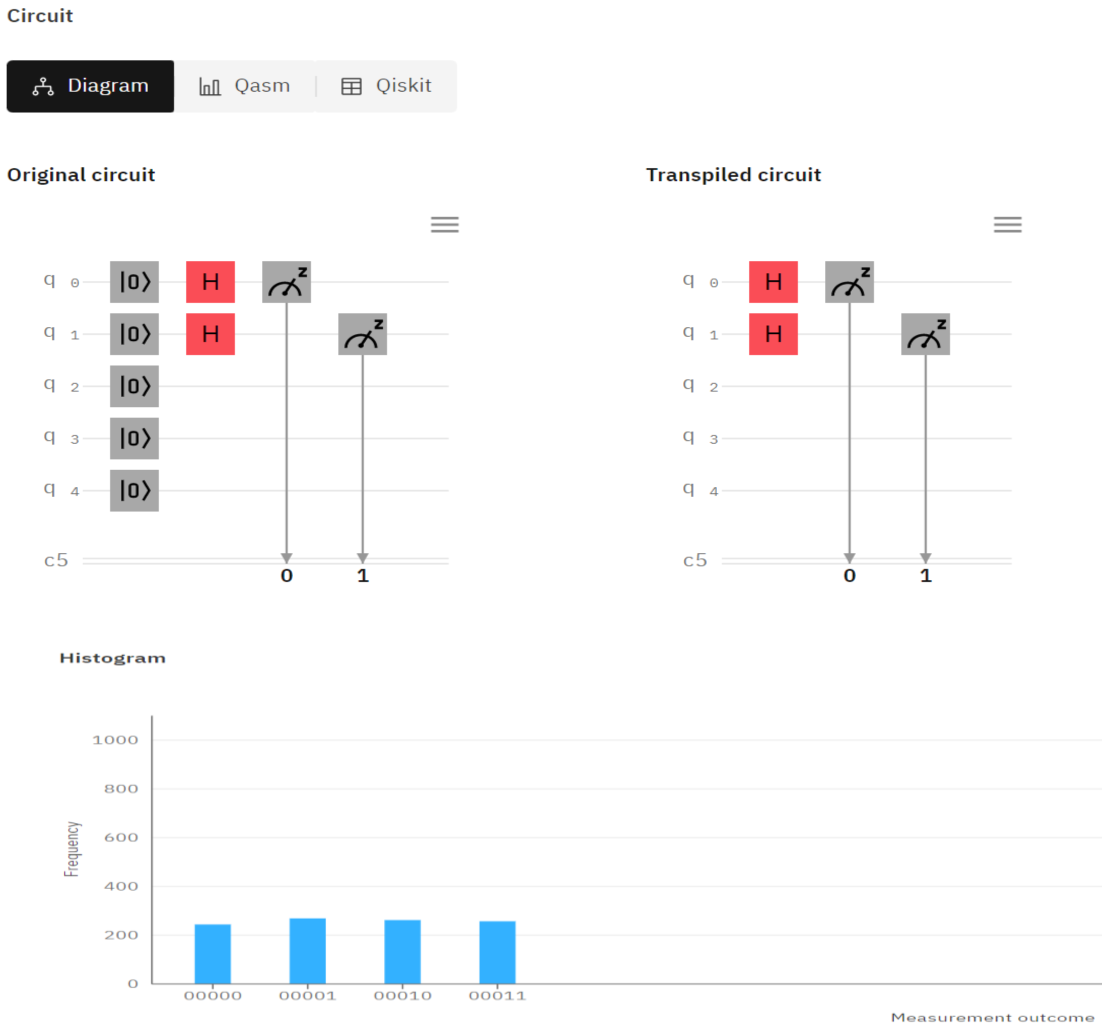
Open in quantum lab

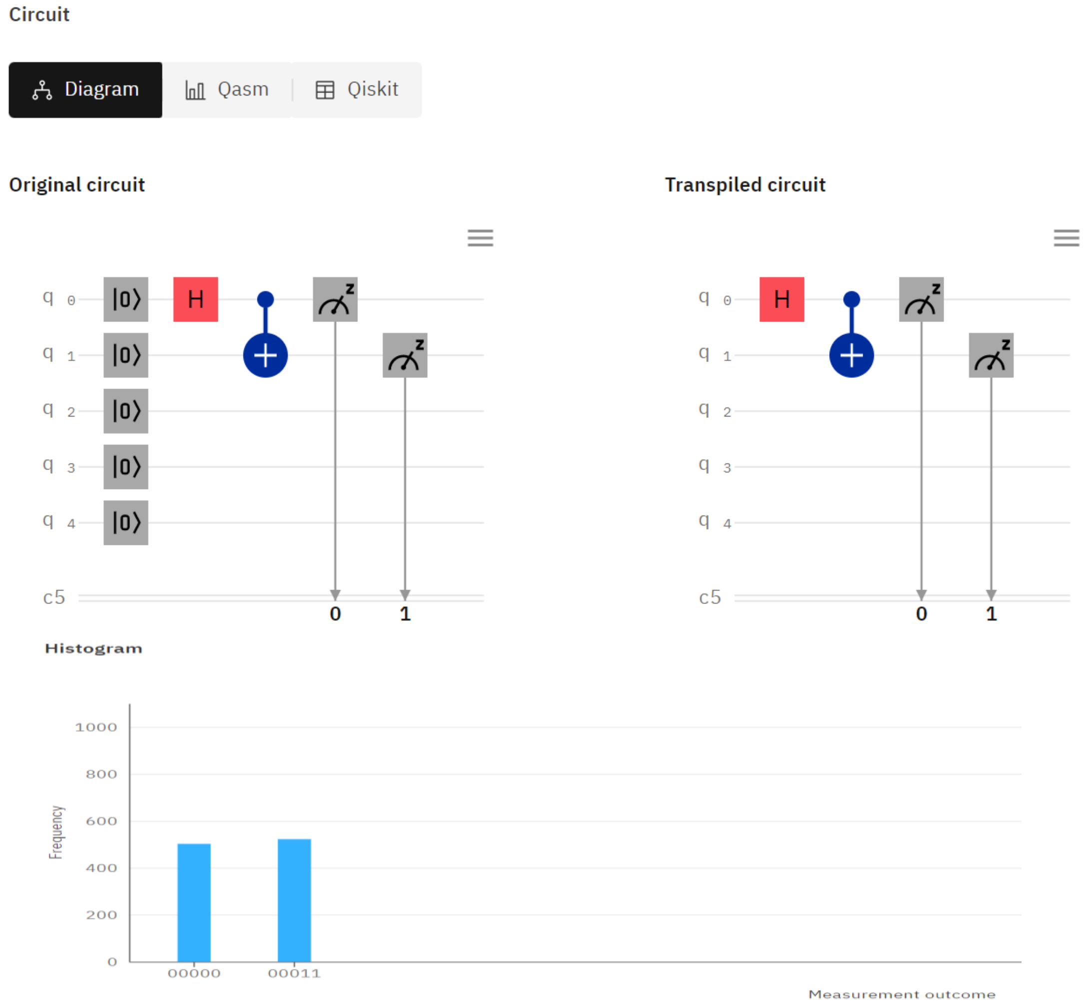
Transpiled circuit

```
from qiskit import QuantumRegister,
ClassicalRegister, QuantumCircuit
from numpy import pi

qreg_q = QuantumRegister(5, 'q')
creg_c = ClassicalRegister(5, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.h(qreg_q[0])
circuit.cx(qreg_q[0], qreg_q[1])
circuit.measure(qreg_q[0], creg_c[0])
circuit.measure(qreg_q[1], creg_c[1])
```

Open in quantum lab

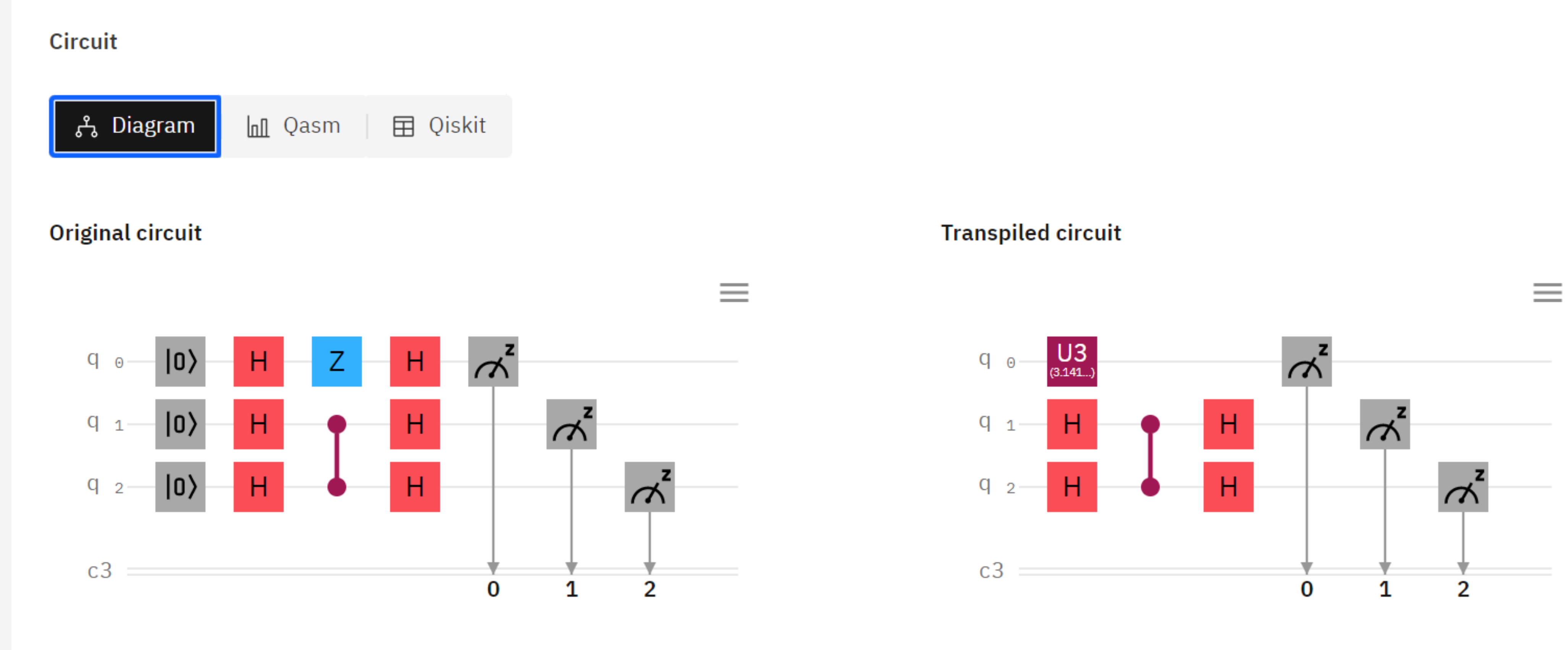
Two qubits in superposition state vs. Entanglement





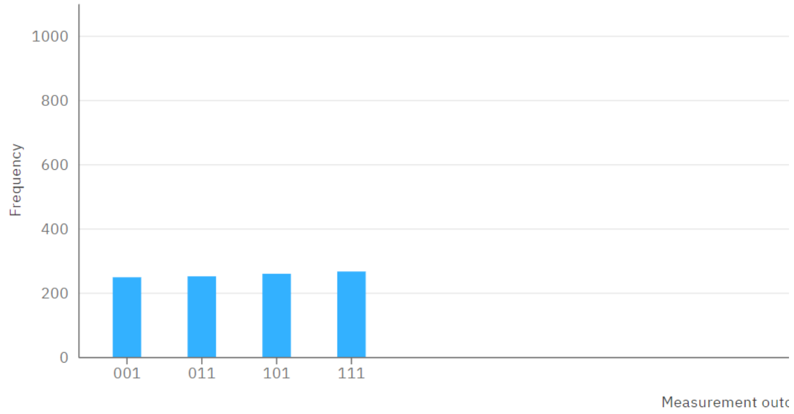
IBM-QASM (Circuit Composer)

- Deutsch-Jozsa Algorithm –Example, N=3 balanced



- Deutsch-Jozsa Algorithm –Example results

Histogram



Circuit

Diagram | Qasm | Qiskit

Original circuit

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[3];
creg c[3];

reset q[0];
reset q[1];
reset q[2];
h q[0];
h q[1];
h q[2];
z q[0];
cz q[1],q[2];
h q[0];
h q[1];
```

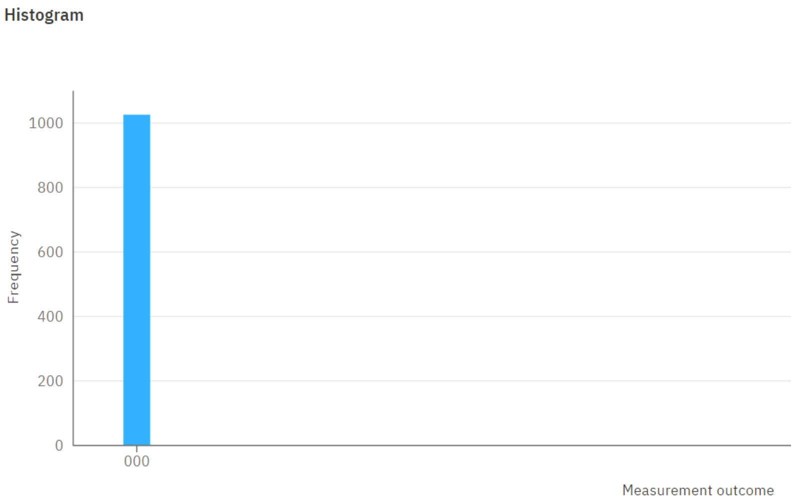
Open in composer

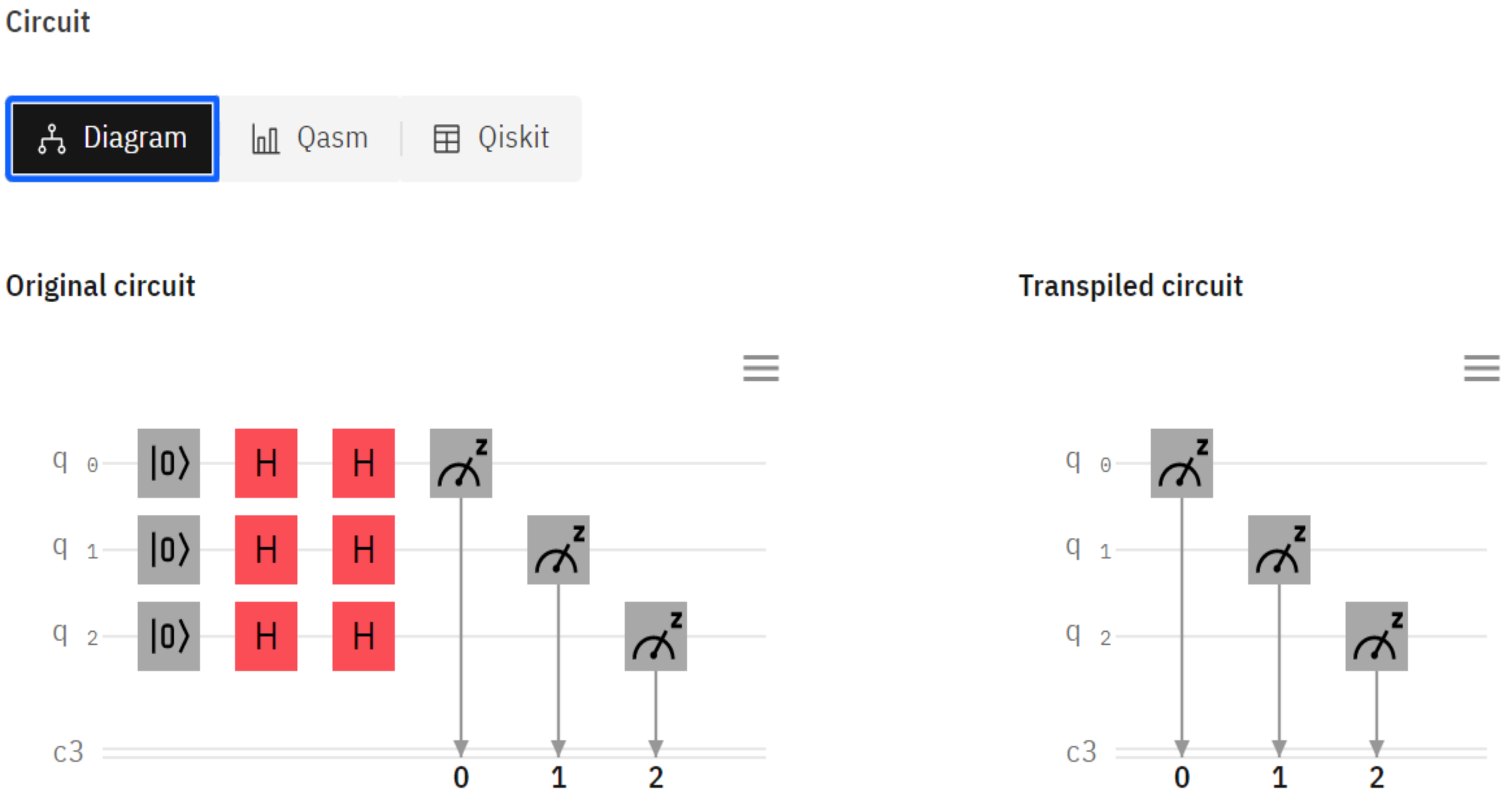
Transpiled circuit

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[3];

creg c[3];


u3(3.141592653589793, -2.220446049250313e-16,
h q[1];
h q[2];
cz q[1], q[2];
h q[1];
h q[2];
measure q[0] -> c[0];
measure q[1] -> c[1];
```

Open in composer

- Deutsch-Jozsa Algorithm– N=3 (Constant)





Quantum Memory and Quantum Computing

(Epilogue)
(updates)

Quantum Memory

- 50-400 qubits Quantum computer may be able to perform tasks which surpass the capabilities of today's classical digital computers, quantum memory is an essential element.
- Qubits' noise will limit the size of quantum circuits – Reliable?
- Quantum Computer is powerful.
- Quantum Computer is hard– Qubits quality

Quantum Memory : Stable and Long Storage time

Quantum Computing in the NISQ era and beyond, John Preskill
Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena CA 91125, USA
30 July 2018

Quantum Simulations

- Better Quantum Simulation– Quantum Algorithm to verify Quantum Computer demonstrates Quantum Advantages over Classical computer with the best algorithms.
- Scientific Opportunities for Quantum Simulators

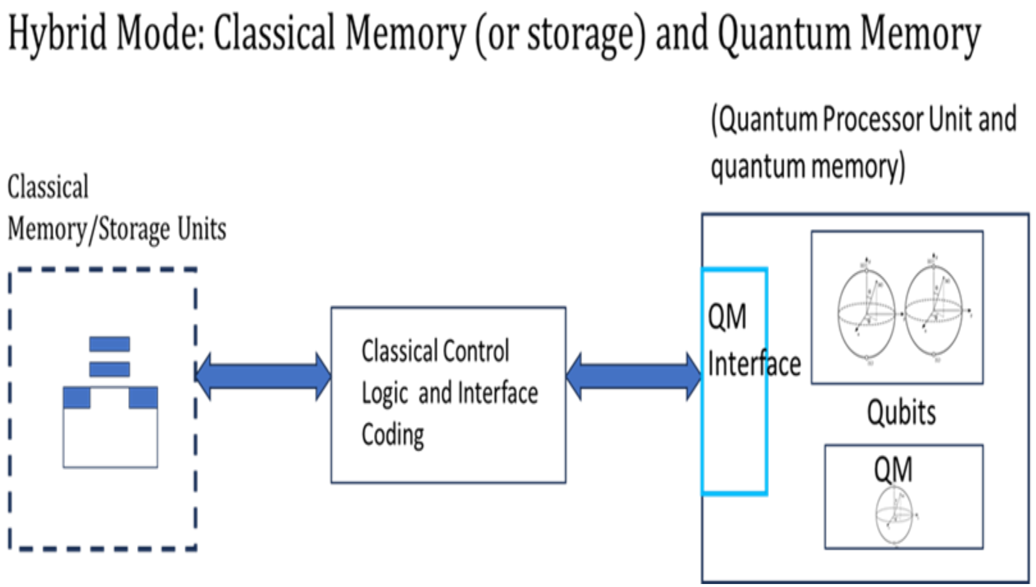
  Quantum material, Quantum chemistry, Quantum devices and transport,

  Gravity, particle physics and cosmology, and Non equilibrium body dynamics

Quantum Memory : Stable and Long Storage time

Applications of Quantum Memory

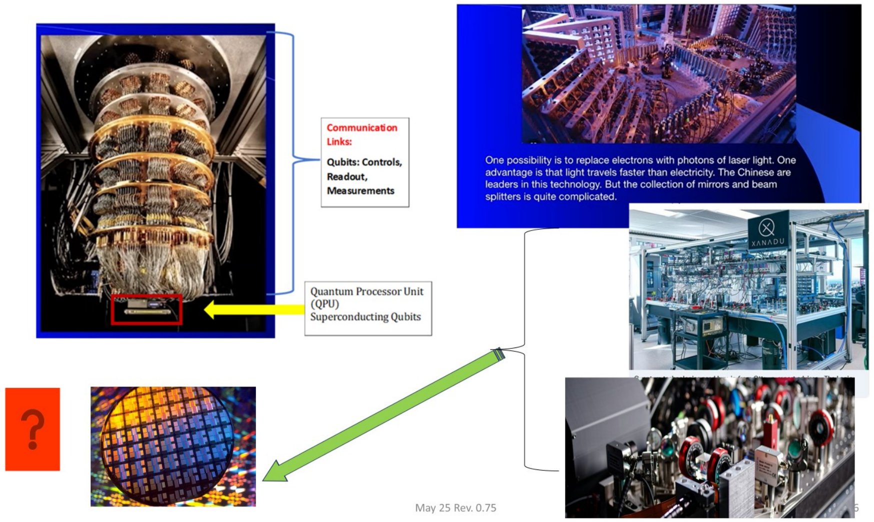
- Quantum computing technology, quantum memory, and memory storage rapidly progressed. The quantum memory limited in quantum register level within QPU
- The wide adaptions of Quantum communication require stable quantum memory and storage
- The no-clone theorem poses a significant challenge to using quantum memory storage to store many quantum states, a key limitation in the field.
- The concept of using classical memory to store large amounts of quantum states and then transferring them to quantum memory is a possibility.



Creating Reliable Qubits---Challenge (task)

Quantum Error Correction?

Engineering resources ---– Engineers



Quantum Computing needs Semiconductor Technology

All Electronics need Semiconductor Chips including Quantum Processor Unit, Communication Unit, and Quantum State Controllers.

Appendix #4 –Notes of Quantum Computer Hardware Design –
Dec. 3, 2023

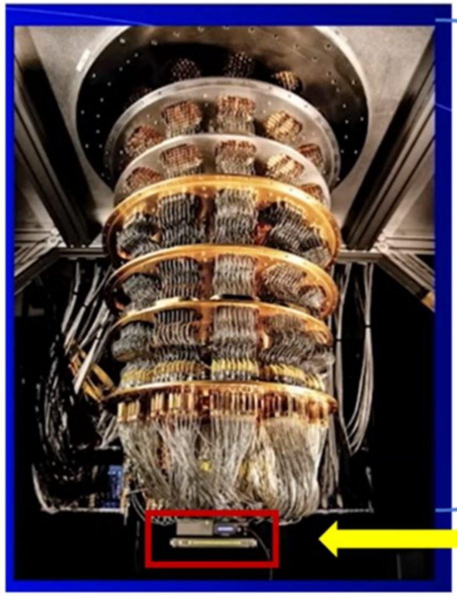
How to Scale up Quantum Computer?

- Large numbers of interconnect/entangled cables of electronic circuits to control and measure Qubits create a bottleneck for Quantum Computer to scale to Large Quantum Computers
- New and stable Qubits and solid or advanced communication to connect the external (room temperature) control and read-out equipment to low-temperature Qubits (Superconductor Qubits).
- The electronic circuits have to be very accurate; many circuits have to be placed to close the low temperatures' qubits; the Qubits require many analog and digital circuits to control Qubits

How to Scale up Quantum Computer? (2)

- QPU operates under Low Temperature (milli Kelvin Temperature)
- Integration of control and readout that maintain Qubit coherence at low temperature
  - Solutions to reduce the number of cables?
  - For million qubits system, it requires millions cables, it is an untannable engineering problems
  - To place readout circuit and control logic inside of the dilution refrigerator to reduce the number of cables.
- 3D integration technology is required

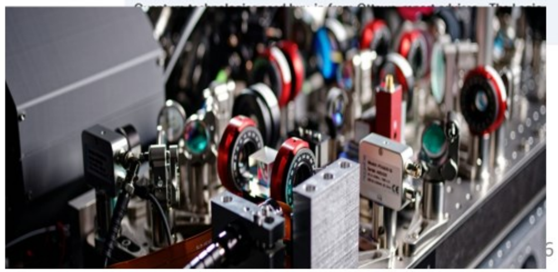
Fundamental Structures of Quantum Computer Hardware Design



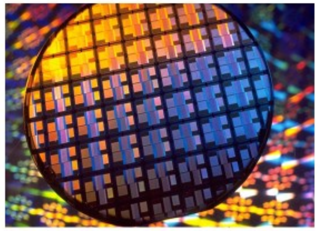
Communication Links:

Qubits: Controls, Readout, Measurements

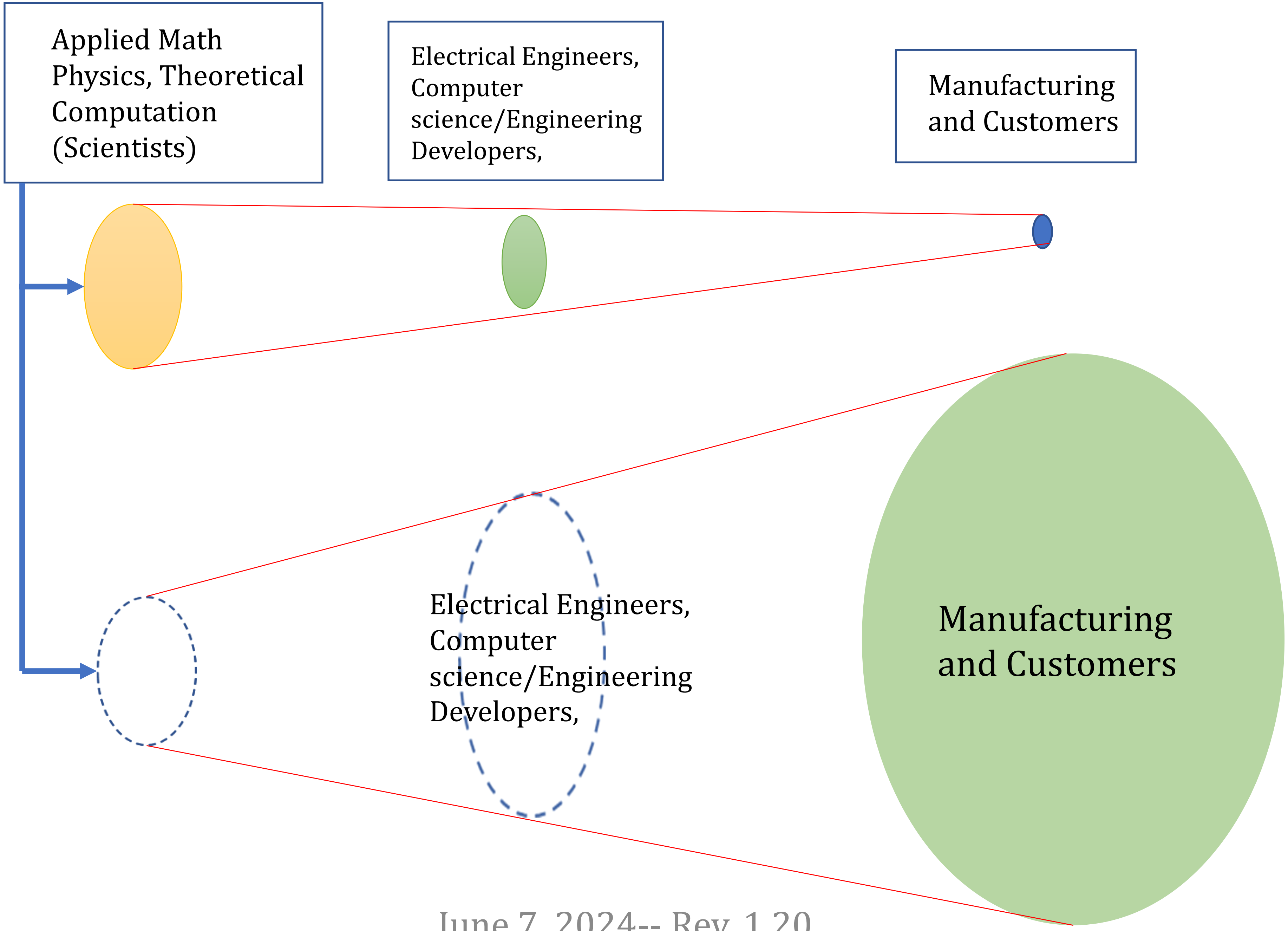
Quantum Processor Unit (QPU)
Superconducting Qubits



?



May 25 Rev. 0.75


Applied Math
Physics, Theoretical
Computation
(Scientists)
Electrical Engineers,
Computer
science/Engineering
Developers,
Manufacturing
and Customers
Electrical Engineers,
Computer
science/Engineering
Developers,
Manufacturing
and Customers

- ❑ Building Quantum Computer hardware has many challenges;

  - ✓ Low Temperature (milli Kelvin Temperature)
  - ✓ Integration of control and readout that maintain Qubit coherence at low temperature
    - ✓ Need advanced communication circuits, MOSFET based circuit.
  - ✓ 3D integration technology is required, and
- ❑ Strong Semiconductor Technology knowledge

Summing Up:

- ❑ Today's Quantum Computer company has three parts of expertise: building semiconductor chips including software, manufacturing the QC hardware (assembling into compact package), and Quantum physics.
- ❑ The semiconductor (chip) company won the market from the mini-computer and later the supercomputer markets because the chip company knew how to produce chips, not because the chip company had the best computer architectures.

Semiconductor Chips!

Reference/Resource:

1. Nielsen and Chung: Quantum Computation and Quantum Information
2. N. David Mermin: Quantum Computer Science
3. Bernard Zygelman: A First Introduction to Quantum Computing and Information
4. P. Krantz, et al: A Quantum Engineer's Guide to Superconducting Qubits
5. Willian D. Oliver: Lecture Notes of the 44^{th} IFF Spring School 2013, Superconducting Qubits
6. Mark Oskin: Quantum Computing-Lecture Notes (Department of Computer Science and Engineering, University of Washington)
7. Ryan O' Donnell: Lecture Notes, Quantum Computation and Quantum Information 2018 (Carnegie Mellon University)
8. Sergory Frolov: Lecture Notes, Quantum Transport, University of Pittsburg
9. Shor's course: Quantum Computing (MIT18.435 / 2.111 : http://www-math.mit.edu/)

Resource:

1. Robert Loredo: Learn Quantum Computing with Python and IBM Quantum Experience
2. Jack D. Hidary: Quantum Computing: An Applied Approach
3. IBM Qiskit: https://qiskit.org/textbook/preface.html
4. Quantum Control System: https://www.zhinst.com/americas/en/quantum-computing-systems/qccs?gclid=CjwKCAjwxo6IBhBKEiwAXSYBsw7_VYkEQX4g-OndYQVzi4B4ivBaWqZXIXobY_9znuKtWX41ppWTQhoCizYQAvD_BwE
5. https://www.dwavesys.com/learn/resource-library/
6. QUANTUM COMPUTING :Progress and Prospects https://www.nap.edu/read/25196/chapter/1#iii

Reference/Resource (2):

[1A] Joseph C. Bardin et al., “Design and Characterization of a 28-nm Bulk-CMOS Cryogenic Quantum Controller Dissipation Less Than 2 mW at 3K”
[2A] R. McDermott et al., “Quantum-Classical Interface Based on Single Flux Quantum Digital Logic”
[3A] E. Leonard Jr. et al., “Digital Coherent Control of a Superconducting Qubit”
[4A] D. Rosenberg et al., “3D integrated superconducting qubits”
[5A] Christian Kraglund Andersen et al., “Repeated Quantum Error Detection in a Surface Code”
[6A]Arnout Beckers et al. Characterization_and_Modeling_of_28nm_Bulk_CMOS_Technology_Down_to_4.2_K
[7A] David J. Frank, et al; A_Cryo-CMOS_Low-Power_Semi-Autonomous_Qubit_State_Controller_in_14nm_FinFET_Technology, ISSCC 2022
[8A] Michael Tinkham, “Introduction to Superconductivity”, Second Edition
[9A] Frederic Parment et al., “Air-Filled SIW for low loss and high power handling Millimeter-Wave Substrate Integrated Circuits”; IEEE Transactions On Micowave Theory and Techniques,, Vol 63, No. 4, April 2015.
[10A] Thomas E. Roth, Ruichao Ma, and Weng C. Chew: An Introduction to the Transmon Qubit for Electromagnetic Engineers, June 2021
[11A] Daniel W. Bliss, “Modern Communications A Systematic Introduction”, Cambridge University Press, 2022

Appendix

Linear Algebra for Quantum Computing—Review

Appendix QT- Math Notes

Deutsch-Jozsa Math Notes

Notes of Quantum Computer Hardware Design

the Future of Memory and Storage

Appendix 1

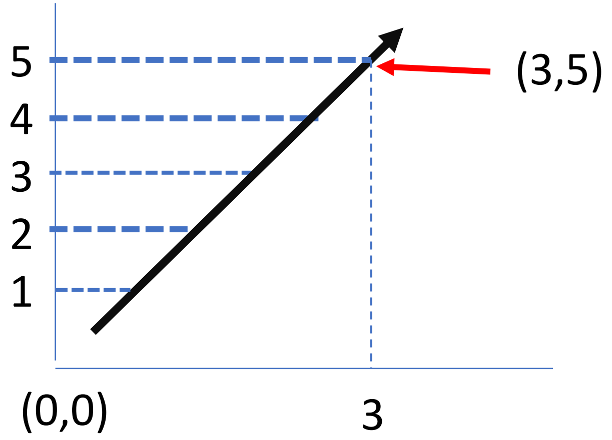
(Linear Algebra for Quantum Computing--Review)

Linear Algebra for Quantum Computing

- Linear Algebra (LA) is the language of Quantum Computing.
- Basic of Linear Algebra

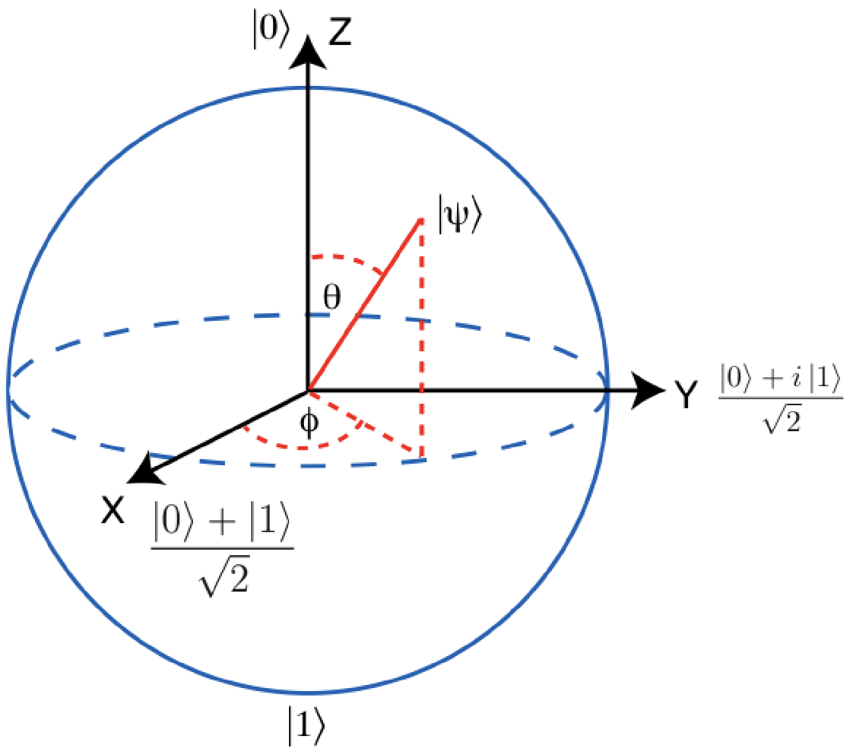
Vector |v>, "is a mathematical quantity with both *direction* and *magnitude*"

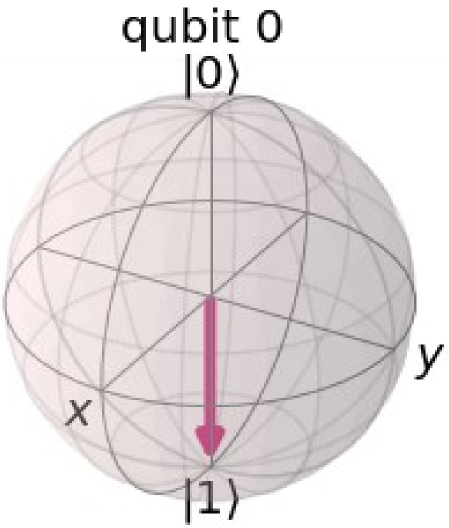
$|v\rangle = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$,



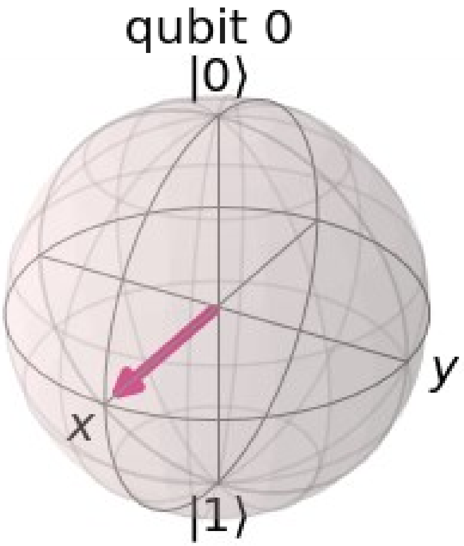
In Quantum Computing, State-vector → corresponds to a specific Quantum State

- Bloch Sphere





Superposition between |0>, |1>. The arrow is halfway between |0>, at the top, |1> at the bottom.
Arrow can rotate to anywhere surface of the sphere.



https://qiskit.org/textbook/ch-states/introduction.html

Vector Space

$\begin{bmatrix} x1 \\ y1 \end{bmatrix} + \begin{bmatrix} x2 \\ y2 \end{bmatrix} = \begin{bmatrix} x1 + x2 \\ y1 + y2 \end{bmatrix}$,

n|ν> = $\begin{bmatrix} nx \\ ny \end{bmatrix}$ ∈ ν, ∀n ∈ R, |ν>=$\begin{bmatrix} x \\ y \end{bmatrix}$

|a> + |b> = |c>

Matrices and Matrix Operation: Matrices are mathematical objects that transform vectors into other vectors,

|ν> → |ν'>= M|ν>

Examples– Matrix Multiplication

Example A

$$AB = \begin{bmatrix} 3 & 4 \\ 1 & 5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 16 \\ 7 & 9 \\ 4 & 8 \end{bmatrix}$$, A rows times B columns

(row 1) x (Column 1) $= \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 10$

(row 2) x (Column 1) $= \begin{bmatrix} 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 7$

(row 3) x (Column 1) $= \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 4$

Example B

$$\begin{bmatrix}2 & 0\\ 5 & -1\end{bmatrix}\begin{bmatrix}-3 & 1\\ 2 & 1\end{bmatrix}=\begin{bmatrix}(2)(-3)+(0)(2) & (2)(1)+(0)(1)\\ (5)(-3)+(-1)(2) & (5)(1)+(-1)(1)\end{bmatrix}$$

$$=\begin{bmatrix}-6 & 2\\ -17 & 4\end{bmatrix}$$

Example C
(Columns times rows)

$$AB=\begin{bmatrix}3\\1\\2\end{bmatrix}[2\ \ 4]+\begin{bmatrix}4\\5\\0\end{bmatrix}[1\ \ 1]=\begin{bmatrix}6 & 12\\ 2 & 4\\ 4 & 8\end{bmatrix}+\begin{bmatrix}4 & 4\\ 5 & 5\\ 0 & 0\end{bmatrix}=\begin{bmatrix}10 & 16\\ 7 & 9\\ 4 & 8\end{bmatrix}$$

- $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow$ Column vector (3x1)

- $[a\ b\ c]$ => Row(1x3) ; $\begin{bmatrix} a\ b\ c \\ d\ e\ f \\ g\ h\ i \end{bmatrix}$ = 3x3 matrix

- $\begin{bmatrix} a\ b\ c \\ d\ e\ f \\ g\ h\ i \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} A1 \\ B2 \\ C3 \end{bmatrix}$; $[a\ b\ c] \begin{bmatrix} a\ b\ c \\ d\ e\ f \\ g\ h\ i \end{bmatrix} = [R1\ R2\ R3]$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} . \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 32 \\ 50 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} . \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = [30\ 36\ 42]$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

We manipulate qubits in Quantum Computer by applying sequences of Quantum Gates. Quantum gates can be expressed as a matrix that can be applied to state vectors.

Ex. $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, Quantum gate: Pauli- X

Computation Basis $|0>, |1>, |0> = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1> = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\sigma_x |0> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (0)(1) + (1)(0) \\ (1)(1) + (0)(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1>$ (flip the state from $|0>$ to $|1>$)

$\sigma_x |1> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0>$

Hermitian and Unitary Matrices

- Hermitian matrix is a matrix that is equal to its *conjugate transpose*

  ($HH^\dagger = I$) , Ex. $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ ==> $\sigma^\dagger = \begin{bmatrix} 0 & -i \\ -(-i) & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \sigma_y$

- Unitary matrix is a matrix such that the inverse matrix is equal to the *conjugate transpose* of the original matrix, A^{-I} Inverse matrix,

  $A^{-I} A = AA^{-I} = I$, Identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Math Note: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ➔ $A^{-I} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ det A = ad – bc

- Example $(A^{\dashv}A = AA^{\dashv} = I)$

$$\sigma_y = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \qquad \sigma_y{}^{\dashv} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_y\ \sigma_y{}^{\dashv} = \sigma_y{}^{\dashv}\sigma_y = \begin{bmatrix} (0)(0)+(-i)(i) & (0)(-i)+(-i)(0) \\ (i)(0)+(0)(i) & (i)(-i)+(0)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathrm{I}$$

- Linear Combination of vectors, $|\upsilon_1>, |\upsilon_2>, \ldots\ldots, |\upsilon_n>$

$$|\upsilon> = f_1|\upsilon_1> + f_2|\upsilon_2> + \ldots\ldots\ldots + f_n|\upsilon_n> = \sum_i fi\,|\upsilon_i>$$

Linear dependent

$$b_1|\upsilon_1> + b_2|\upsilon_2> + \ldots\ldots\ldots + b_n|\upsilon_n> = 0$$

At least one of the b_i coefficients is non-zero.

$$\sum_i^n b_i|v_i> = ba|v_a> + \sum_{i,i=a}^n b_i|v_i> = 0$$

$$\Rightarrow \quad |\upsilon_a> = -\sum_{i,i\neq a}^n \frac{b_i}{b_a}|\upsilon_i> = \sum_{i,i\neq a}^n C_i\,|v_i> \Rightarrow |v_a>$$ is a Null Vector .

Ex. $|a> = \begin{bmatrix}1\\0\end{bmatrix}$ & $|b> = \begin{bmatrix}2\\0\end{bmatrix}$, Linear combination, $2|a> - |b> = 0$

Quantum Computation

Basis, $|0\rangle, |1\rangle$

$\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ (linear combination)

Superposition of $|0\rangle$ and $|1\rangle$ basis state, equal probability of measuring the state to be in either one of the basis vectors states, $\frac{1}{\sqrt{2}}$

Hilbert space, Inner product, $|a\rangle, |b\rangle$ -- Inner product: $\langle a|b\rangle$

$\langle a|$ is the conjugate transpose of $|a\rangle$, $|a\rangle^\dagger$

$$\langle a|b\rangle = [a_1^* \; a_2^* \; \ldots\ldots\ldots \; a_n^*]\begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{bmatrix} = a_1^* b_1 + a_2^* b_2 + \ldots\ldots\ldots + a_n^* b_n$$, Where * = complex conjugate

Ex. $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\langle 0|0\rangle = 1$, $\langle \psi|\psi\rangle = 1$

- Unitary Matrix, $U^{\dagger}U = I$

$$|\psi'> = U|\psi>$$

Ex1. $|\psi> a|0> + b|1>$, $U= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,

$|\psi'>=U|\psi> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix} = b|0> + a|1>$

Ex2. Let $|\psi>= 1|0> +0|1> = |0>$, $U=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$|\psi'> =U|\psi>= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0> + \frac{1}{\sqrt{2}}|1>$

Ex3. $U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ then $U^{\dagger} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$U^{\dagger}\ U = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = I$

- Tensor Product ➔ |ϕ> ⊗ |φ> -- tensor product, |ϕ>|φ>

Ex. |ϕ> $=\begin{bmatrix} 2 \\ 6i \end{bmatrix}$, |φ> = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, <ϕ|φ> = $\begin{bmatrix} 2 & -6i \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$=6-24$i$

➔|ϕ>⊗|φ> => |ϕ>|φ>

Ex. |ϕ>|φ> $=\begin{bmatrix} 2 \\ 6i \end{bmatrix} \otimes \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \times 3 \\ 2 \times 4 \\ 3 \times 6i \\ 4 \times 6i \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 18i \\ 24i \end{bmatrix}$

A* -- complex conjugate of matrix A::

If A=$\begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix}$; A* = $\begin{bmatrix} 1 & -6i \\ -3i & 2-4i \end{bmatrix}$;

A[⊺] => transpose of matrix A

$A^{\intercal} = \begin{bmatrix} 1 & 3i \\ 6i & 2+4i \end{bmatrix}$

A† -- Hermitian Conjugate (adjoint) of matrix A

If A= $\begin{bmatrix} 1 & 6i \\ 3i & 2+4i \end{bmatrix}$, $A^{\dagger} = \begin{bmatrix} 1 & -3i \\ -6i & 2-4i \end{bmatrix}$

Note: $A^{\dagger} = (A^*)^T$

Math Note Box:

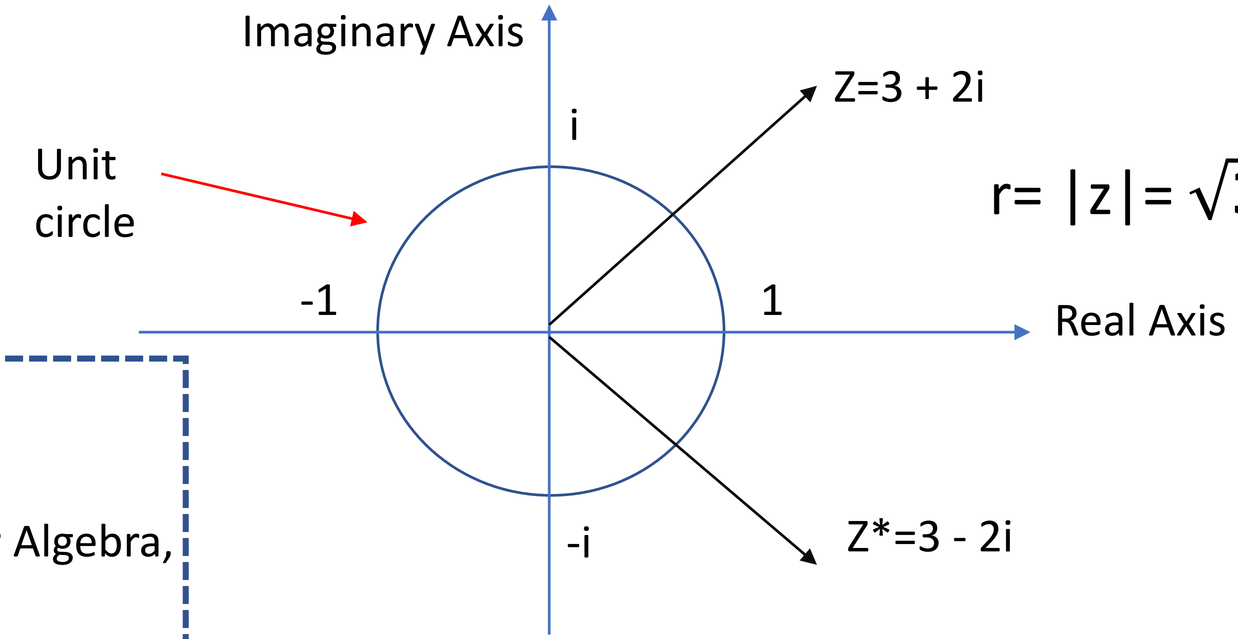
Math Note:

The complex conjugate of 3+2i is 3-2i, “-, bar” =“ * “

The complex conjugate of $z = 1 - i$ is $z^* = 1 + i$

In general, $z = a + bi$, $z^* = a - bi$

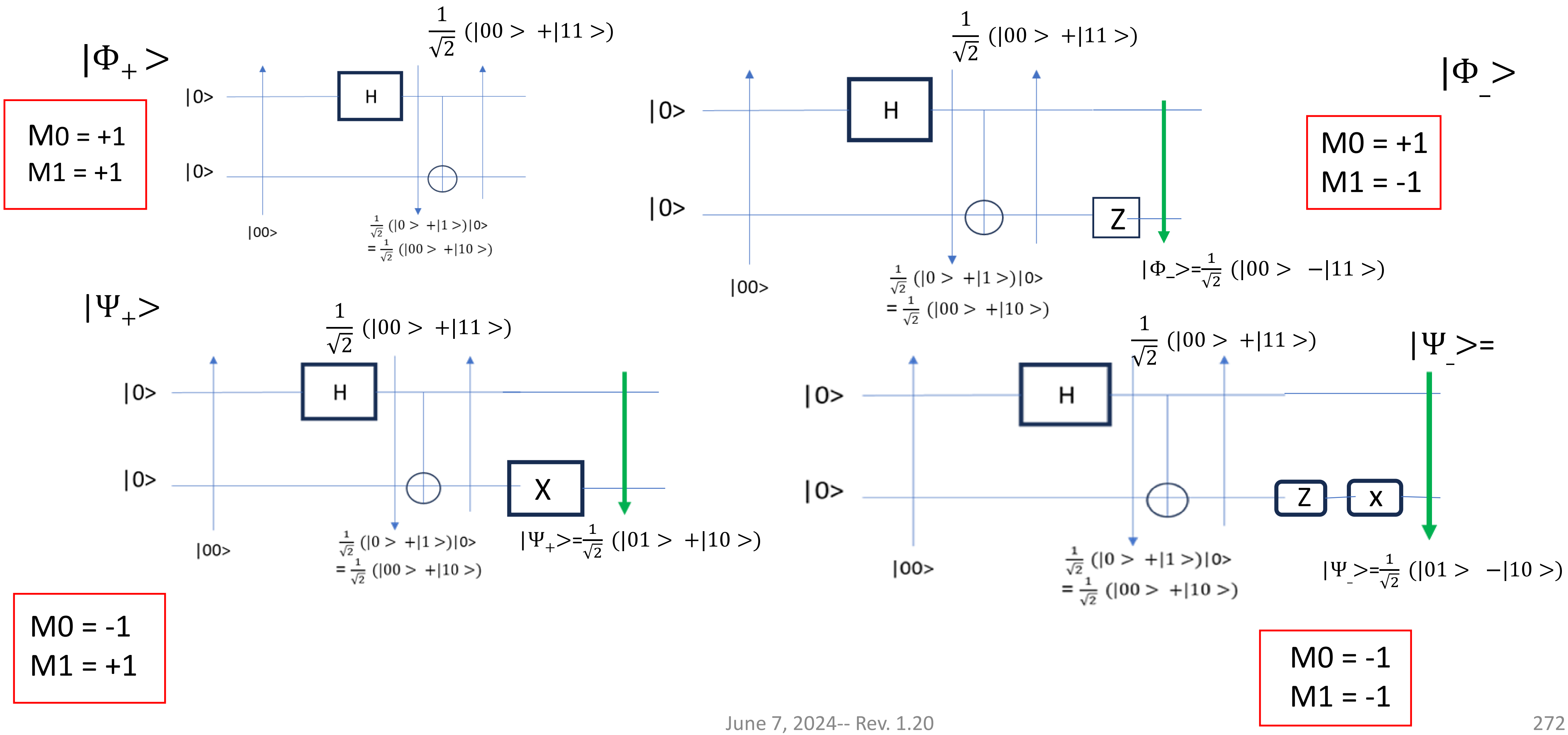
Complex Plane:



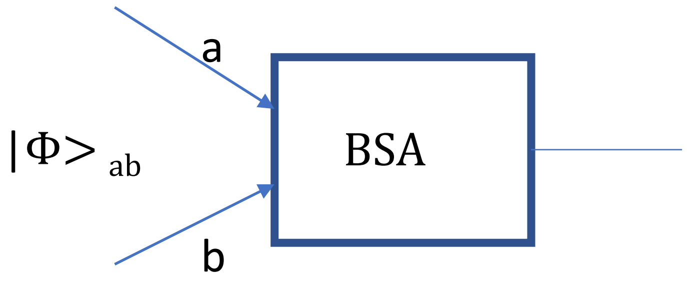
$r = |z| = \sqrt{3^2 + 2^2}$

Reference:
Gilbert Strang,
Introduction to Linear Algebra,
Fifth Edition

Preparing Bell States:

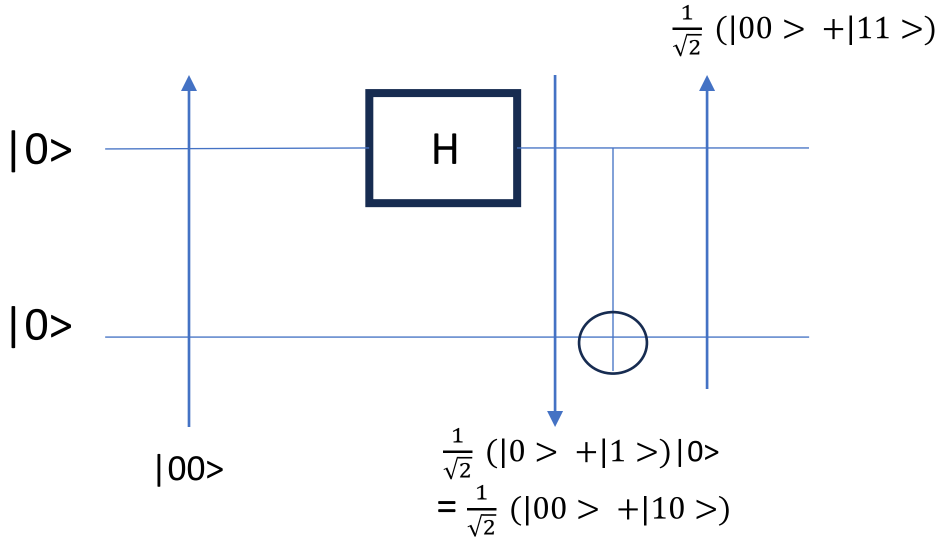


Conceptual Bell State Analyzer



Bell States:

$$|\Phi_+ > = \frac{1}{\sqrt{2}} (|00 > \ +|11 >)$$
$$|\Phi_- > = \frac{1}{\sqrt{2}} (|00 > \ -|11 >)$$
$$|\Psi_+ > = \frac{1}{\sqrt{2}} (|01 > \ +|10 >)$$
$$|\Psi_- > = \frac{1}{\sqrt{2}} (|01 > \ -|10 >)$$

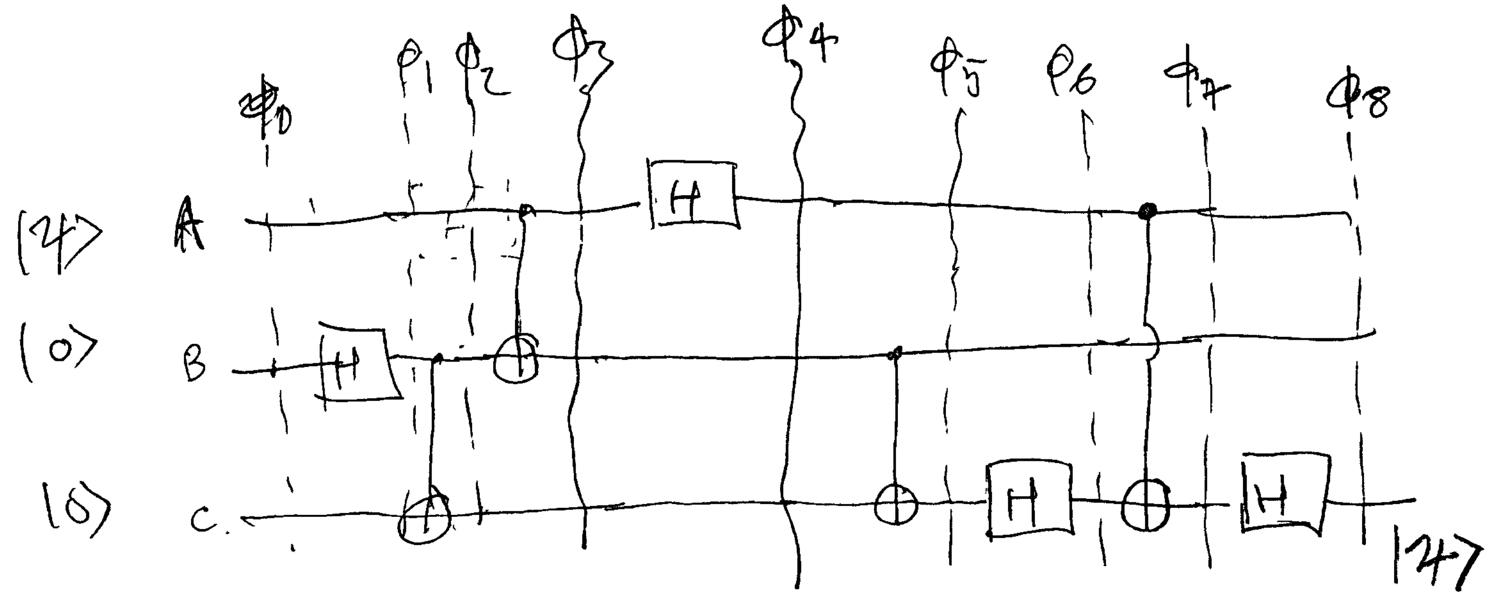


(Entanglement)

Appendix 2

Math Notes for Teleportation

the Future of Memory and Storage



$$|\psi\rangle \otimes |0\rangle \otimes |0\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle \otimes |0\rangle = |\psi 00\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1.$$

$$\Phi_0 = |\psi 00\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle \otimes |0\rangle$$

$$= \alpha|000\rangle + \beta|100\rangle$$

$$\Phi_1 = H_2|\psi 00\rangle = \alpha \otimes H|0\rangle \otimes |0\rangle + \beta|1\rangle \otimes H|0\rangle \otimes |0\rangle$$

$$= \alpha|0\rangle \otimes \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes |0\rangle +$$

~~$+ \beta|1\rangle \otimes H|0\rangle \otimes$~~

$$+ \beta|1\rangle \otimes \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes |0\rangle$$

$$\Phi_2 \overset{CNOT}{=} \frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|010\rangle\right] + \frac{1}{\sqrt{2}}\left[\beta|100\rangle + \beta|110\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle\right] + \frac{1}{\sqrt{2}}\left[\beta|100\rangle + \beta|111\rangle\right]$$

$$\Phi_3 \overset{CNOT\,on\,2}{=} \frac{1}{\sqrt{2}}\left[\alpha|000\rangle + \alpha|011\rangle\right] + \frac{1}{\sqrt{2}}\left[\beta|110\rangle + \beta|101\rangle\right]$$

$\phi_4 =$ Hadamard gate on bit.

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left[\alpha(|0\rangle+|1\rangle)|0\rangle|0\rangle + \alpha(|0\rangle+|1\rangle)|1\rangle|1\rangle\right.$$

$$\left. + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\left[\beta(|0\rangle-|1\rangle)|1\rangle|0\rangle + \beta(|0\rangle-|1\rangle)|0\rangle|1\rangle\right]\right]$$

$$= \frac{1}{2}\left[\alpha(|000\rangle+|100\rangle) + \alpha(|011\rangle+|111\rangle)\right]$$

$$+ \frac{1}{2}\left[\beta(|010\rangle-|110\rangle)\right] + \beta(|001\rangle-|101\rangle)\Big]$$

$\phi_5 \underset{=}{\text{App CNOT}} ⓷$

$$= \frac{1}{2}\left[\alpha(|000\rangle+|100\rangle+|010\rangle+|110\rangle\right]$$

$$+ \frac{1}{2}\left[\beta(|011\rangle - |111\rangle + |001\rangle - |101\rangle\right]$$

$\phi_6 \underset{=}{\text{App. H}} ⓷$

$$= \frac{1}{2}\frac{1}{\sqrt{2}}\left[\alpha(|00\rangle\otimes(|0\rangle+|1\rangle) + |10\rangle\otimes(|0\rangle+|1\rangle) + |01\rangle\otimes(|0\rangle+|1\rangle)\right.$$

$$\left. + |11\rangle\otimes(|0\rangle+|1\rangle)\right]$$

$$+ \frac{1}{2}\frac{1}{\sqrt{2}}\left[\beta(|01\rangle\otimes(|0\rangle-|1\rangle) - |11\rangle\otimes(|0\rangle-|1\rangle) + |00\rangle\otimes(|0\rangle-|1\rangle)\right.$$

$$\left. - |10\rangle\otimes(|0\rangle-|1\rangle)\right)\Big]$$

$$\phi_{6,2} = \frac{1}{2}\alpha\left[|000\rangle + |001\rangle + |100\rangle + |101\rangle + |010\rangle + |011\rangle + |110\rangle + |111\rangle\right]$$

$$+ \frac{1}{2}\beta\left[|010\rangle - |011\rangle - |110\rangle + |111\rangle + |000\rangle - |001\rangle - |100\rangle + |101\rangle\right]$$

$$\phi_7 \overset{\text{bit 3 CNOT}}{=} \frac{1}{2}\alpha\left[|000\rangle + |001\rangle + |101\rangle + |100\rangle + |010\rangle + |011\rangle + |111\rangle + |110\rangle\right]$$

(a)

$$+ \frac{1}{2}\beta\left[|010\rangle - |011\rangle - |111\rangle + |110\rangle + |000\rangle - |001\rangle - |101\rangle - |100\rangle\right]$$

(b)

$$\phi_8 \overset{H \text{ on } ③}{=} \frac{1}{2}\alpha\left[|00\rangle\otimes(|0\rangle + |1\rangle) + |00\rangle\otimes(|0\rangle - |1\rangle) + |10\rangle\otimes(|0\rangle - |1\rangle) + |10\rangle\otimes(|0\rangle + |1\rangle) + |01\rangle\otimes(|0\rangle + |1\rangle) + |01\rangle\otimes(|0\rangle - |1\rangle) + |11\rangle\otimes(|0\rangle - |1\rangle)\right] + |11\rangle\otimes(|0\rangle + |1\rangle)\Big]$$

$$= \frac{1}{2}\alpha\left[|000\rangle + |001\rangle + |000\rangle - |001\rangle + |100\rangle - |101\rangle + |100\rangle + |101\rangle + |010\rangle + |011\rangle + |010\rangle - |011\rangle + |110\rangle - |111\rangle + |110\rangle + |111\rangle\right]$$

$$= \frac{\alpha}{2}\left[|000\rangle + |100\rangle + |010\rangle + |110\rangle\right] + \frac{\beta}{2}\left[|011\rangle + |111\rangle + |001\rangle + |101\rangle\right]$$

$$= \frac{1}{2}\left[|00\rangle + |10\rangle + |01\rangle + |11\rangle\right]\otimes\alpha|0\rangle + \frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle\right]\otimes\beta|1\rangle$$

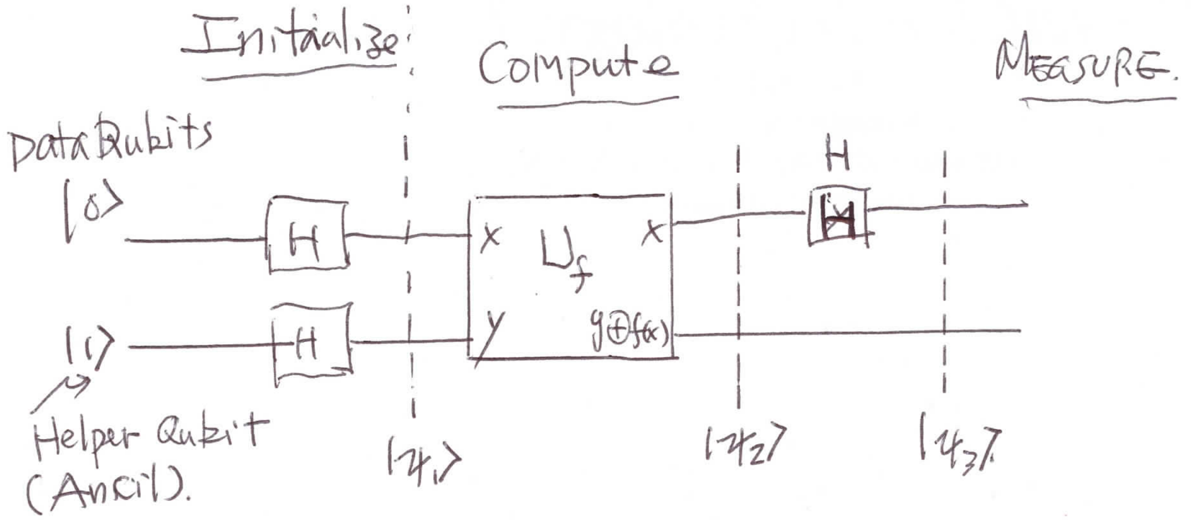
$$= \frac{1}{2}\left[\quad\right]\otimes(\alpha|0\rangle + \beta|1\rangle)$$

$$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle)$$

Appendix 3

Deustch- Josa Problem Math Notes

the Future of Memory and Storage



INPUT : $|\psi_0\rangle = |0, 1\rangle$

Apply the H-Gate to the query and result equation.

$$|\psi_1\rangle = H^{\otimes}|01\rangle$$
$$= H|01\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Examine $y \oplus f(x)$

Suppose $f(x) = 0$ then $y \oplus f(x) = y \oplus 0 =$

"$\oplus$ = XOR"

$$\frac{1}{\sqrt{2}}(|0 \oplus 0\rangle - |1 \oplus 0\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Suppose $f(x) = 1$ then $y \oplus f(x) = y \oplus 1 =$

$$\frac{1}{\sqrt{2}}(|0 \oplus 1\rangle - |1 \oplus 1\rangle) = \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle)$$
$$= \frac{1}{\sqrt{2}}(-|0\rangle + |1\rangle)$$

$$y \oplus f(x) = (-1)^{f(x)} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

U_f transforms $\longrightarrow |x\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$(-1)^{f(x)} |x\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$U_f\left[\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)\right] = U_f \frac{1}{2}\left[|0\rangle(|0\rangle-|1\rangle)+|1\rangle(|0\rangle-|1\rangle)\right]$$

$\longrightarrow$ Rearrange.

(A) $\dashrightarrow$ $|\psi_2\rangle$

$$= \frac{1}{2}\left[(-1)^{f(0)}|0\rangle(|0\rangle-|1\rangle) + (-1)^{f(1)}|1\rangle(|0\rangle-|1\rangle)\right]$$

Suppose, $f(0)=f(1)$, $f(x)$ is CONSTANT.

Eq. (A) $\quad = \frac{1}{2}(-1)^{f(0)}\left[|0\rangle(|0\rangle-|1\rangle)+|1\rangle(|0\rangle-|1\rangle)\right]$

$f(0)=\begin{Bmatrix}0\\ \text{or}\\ 1\end{Bmatrix}$

$$= \pm\frac{1}{2}\left[|0\rangle(|0\rangle-|1\rangle)+|1\rangle(|0\rangle-|1\rangle)\right]$$

$$= \pm\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle) \longrightarrow (C)$$

Suppose, $f(0) \neq f(1)$, f is balanced

$$\text{Eq.(A)} = \frac{1}{2}\left[(-1)^{f(0)}|0\rangle(|0\rangle-|1\rangle)+(-1)^{f(1)}|1\rangle(|0\rangle-|1\rangle)\right]$$

$$= \frac{1}{2}\left[(-1)^{f(0)}|0\rangle(|0\rangle-|1\rangle)+(-1)(-1)^{f(0)}|1\rangle(|0\rangle-|1\rangle)\right]$$

Four cases

$f(0)$	$f(1)$
0	0
[0	1]
[1	0]
1	1

$f(0) \neq f(1)$: $f(0)=0$, $f(1)=1$ or $f(0)=1$, $f(1)=0$.

$$(-1)^{f(1)} = (-1)(-1)^{f(0)}$$

$$= \frac{1}{2}(-1)^{f(0)}\left[|0\rangle(|0\rangle-|1\rangle)-|1\rangle(|0\rangle-|1\rangle)\right]$$

$f(0)=\begin{Bmatrix}0\\ \text{or}\\ 1\end{Bmatrix}$

$$= \pm\frac{1}{2}\left[|0\rangle(|0\rangle-|1\rangle)-|1\rangle(|0\rangle-|1\rangle)\right]$$

$$= \pm\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle) \cdots\cdots \to \text{(d)}$$

Apply H on $|x\rangle \longrightarrow$ get $|\psi_3\rangle$

ⓐ $\to |\psi_3\rangle = \pm\frac{1}{\sqrt{2}}|0\rangle(|0\rangle-|1\rangle)$, if $f(0)=f(1)$

ⓑ $\xrightarrow{\text{or}}$ $= \pm\frac{1}{\sqrt{2}}|1\rangle(|0\rangle-|1\rangle)$, if $f(0)\neq f(1)$

Verify ⓐ

From ⓒ $\pm\frac{1}{2}\left[|00\rangle - |01\rangle + |10\rangle - |11\rangle\right]$

⟶ Apply H-Gate, H, on First qubit $|x\rangle$.

$$\pm\frac{1}{2}\overset{H}{H}\left[|00\rangle - |01\rangle + |10\rangle - |11\rangle\right]$$

$$= \pm\frac{1}{2}\left[\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle - \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|1\rangle + \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)|0\rangle - \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)|1\rangle\right]$$

$$= \pm\frac{1}{2\sqrt{2}}\left[|00\rangle + |10\rangle - |01\rangle - |11\rangle + |00\rangle - |10\rangle - |01\rangle + |11\rangle\right]$$

$$= \pm\frac{1}{2\sqrt{2}}\left[2|00\rangle - 2|01\rangle\right] = \pm\frac{1}{\sqrt{2}}|0\rangle\left(|0\rangle - |1\rangle\right) \sim ⓐ$$

#

if $f(0) = f(1)$

Verify (b)

From (d) $= \pm\frac{1}{2}[|00\rangle - |01\rangle - |10\rangle + |11\rangle]$

→ Apply H-Gate, H_1 on First Qubit, $|x\rangle$.

$$= \pm\frac{1}{2}\left[\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|0\rangle - \left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|1\rangle - \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|0\rangle + \left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|1\rangle\right]$$

$$= \pm\frac{1}{2\sqrt{2}}\left[|00\rangle + |10\rangle - |01\rangle - |11\rangle - |00\rangle + |10\rangle + |01\rangle - |11\rangle\right]$$

$$= \pm\frac{1}{2\sqrt{2}}\left[2|10\rangle - 2|11\rangle\right] = \pm\frac{1}{\sqrt{2}}|1\rangle(|0\rangle - |1\rangle) \rightsquigarrow (b) \ \#$$

if $f(0) \neq f(1)$.

IN THIS CASE, $f(0) \oplus f(1) = 0 \Leftrightarrow f(0) = f(1)$

$$|\psi_3\rangle = \pm|f(0) \oplus f(1)\rangle\left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]$$

$f(0) = f(1)$: CONSTANT $\quad f(0) \oplus f(1) \longrightarrow |0\rangle$

$f(0) \neq f(1)$: BALANCED $\quad f(0) \oplus f(1) \longrightarrow |1\rangle$.

Appendix 4

a. Notes of Quantum Computer Hardware Design – Dec. 30, 2023
b. How to Scale up? – Dec. 22, 2023

the Future of Memory and Storage

Notes of Quantum Computer Hardware Design

--- Semiconductor and Quantum Computer Hardware Design

Building Quantum Computer hardware has many challenges;

Low Temperature (milli Kelvin Temperature)

Integration of control and readout that maintain Qubit coherence at low temperature

3D integration technology is required, and

Strong Semiconductor Technology knowledge

Fundamental Structures of Quantum Computer Hardware Design:

Quantum Computer Hardware structures have three function blocks (Figs. 1, 2, and 3).

a. Quantum Processor Unit (QPU) consists of Qubits silicon and other elements.
b. Communication Links: Qubits Controls, Readout, Measurements etc.
The links operated under low temperature to room temperature, and
c. External (room temperature) control units and computers, etc. (Quantum State Controller)

Building Quantum Computer hardware has many challenges;

- Low Temperature (milli Kelvin Temperature)
- Integration of control and readout that maintain Qubit coherence at low temperature
- 3D integration technology is required

Building a large-scale Quantum Computer requires a team of engineers with different technical skills and knowledge of electrical circuits (Analog MOS circuit design for modern comminutions). FPGA, Digital control logic design, and firmware engineers. Organizing a team of engineers with a Quantum Computing and Quantum information background is challenging. It would be best to have an EE in charge of the design but with a physicist or two available to ensure his designs will work.

Today, Quantum Physicists have achieved many results in Quantum computing and information. We need more engineers (I need more engineers) to build the Qubits, hardware, and software. To create a new industry field—Quantum Computing- We need more Electrical and Computer Engineering Engineers. We need experienced Semiconductor engineers to dive into this new field.

This QC design example uses Superconducting Qubits. The superconductor is a well-studied technology. Superconductor Qubit is an excellent candidate for Quantum Computer design. Quantum Processor Units (QPU): Superconducting Qubits demonstrate gate fidelity and coherence time for Quantum computer design. Quantum Computers require many Qubits, and all the Qubits require high fidelity and low variability. Quantum State Controller (QSC) requires many digital and analog circuits, which creates power/scale issues.

In recent years, Quantum Computers scaled from five Qubits to a few hundred Qubits. Superconductor Qubits is one of the leading Quantum modalities compared to other Qubits technologies in terms of scalability and the number of Qubits to build Quantum Computer hardware.

Quantum Computer Hardware Design and Semiconductor Technology:

--Quantum Computer company must operate like a Fabless semiconductor company.

Quantum computer hardware needs to scale up to millions of Qubits for general purpose Quantum Computers. The Quantum Computer manufacturers/developers need to have strong semiconductor technology support. Quantum Computer manufacturers/developers' success depends entirely on the semiconductor technology to produce reliable qubits with higher wafer yields to achieve commercial success. The QPU has to meet standard commercial silicon's requirements. ESD and I/O pin buffers are always supported in commercial silicon chips for protection and signal conversions.
For example, the memory cells never directly contact external signal pins for memory silicon.

Quantum Processor Unit silicon also requires built-in on-chip protection circuits or at least external protections. The Quantum Computer manufacturer requires a team of semiconductor engineers to develop the process technology for Quantum computer applications.

Quantum Computer manufacturers must operate like fabless semiconductor companies; the manufacturers must have the semiconductor engineers develop the wafer process flow, share with the foundry, devices, and circuits engineers, and evaluate the wafer or die electrical characteristics. Quantum Computer company has to act like a classical memory company without fab, but the company must have all level knowledge to produce the Qubits and Quantum controller chips, etc. The Quantum computer company has to have the ability to analyze wafer and die yield results and how to improve the wafer yields. At this stage, foundry cannot offer their Quantum Computer company the good-die wafers. The foundry will not be able to perform any low-yield wafers analysis. The Quantum Computer company has to have a system to handle all the technical issues from wafers, dies, packaged dies, and PCBs. The Quantum Company (QC) has to operate like a Fabless Semiconductor Company. Quantum computer companies must refrain from buying wafers from the foundry to expect the wafer or die to be thoroughly tested and mounted on PCB as QPU. Unlike the matured products, such as logic (ASIC, CPU) and memory, the foundry can handle complete services from wafer sort, package, and final products to the QC companies.

Today's foundry process/technology targets *logic* and *memory* processes, so making changes is challenging. QC customers can only expect the foundry to provide some of the services like the matured semiconductor company.

Quantum Computer hardware's Communication links (Fig. 4):
All components require analog MOS circuit design for modern comminutions and require high accuracy and low power. All signals must have low clock signal skew and minimum race conditions between signals. Allowing thousands of signals from the Quantum Controller to reach QPU's Qubits is a challenging engineering task for analogy circuit designs. Qubits have coherence time limitations. Unlike digital circuits, which have many circuit techniques to control all the signals to reach the targeted CPU or memory chips simultaneously, Fig 1, Fig 3, and Fig 4. The discussed semiconductor knowledge is in Engineering class 101. Implementing the complex system's hardware designs is a challenging design project, requiring experienced design engineers and fab engineers. Quantum Company requests experienced manufacturing engineers to assemble the parts and pack them into one system. Classical computers have much better expertise in packing all the pieces of Quantum Computer hardware into a compact size.

Summing Up:

Today's Quantum Computer company has three parts of expertise: building semiconductor chips including software (software), manufacturing the QC hardware (assembling into compact package), and Quantum physics.
The semiconductor (chip) company won the market from the mini-computer and later the supercomputer markets because the chip company knew how to produce chips, not because the chip company had the best computer architectures.

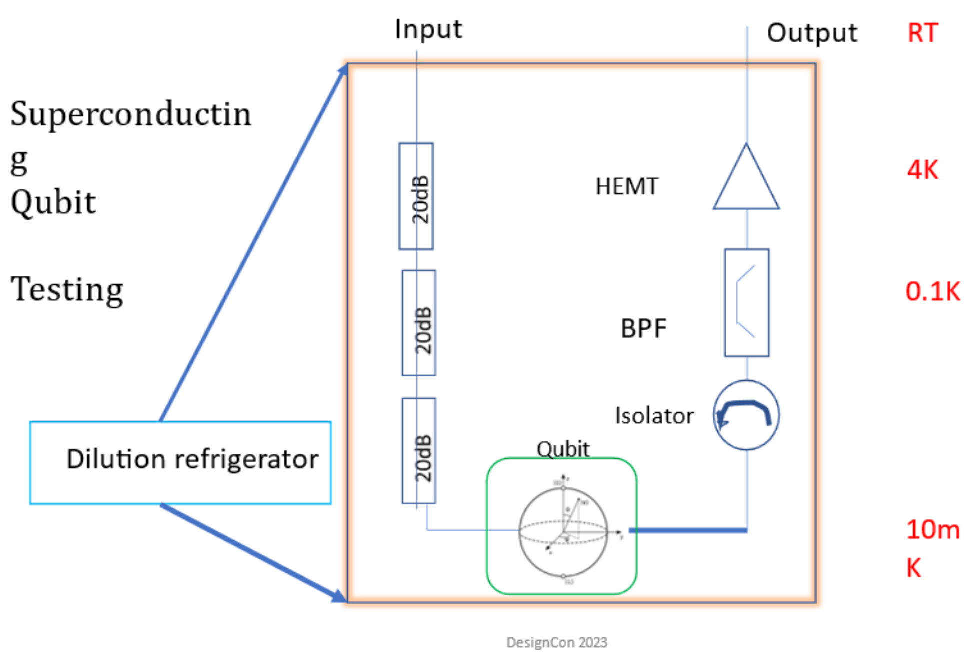


Fig 1 Block diagram of Dilution refrigerator with analog components

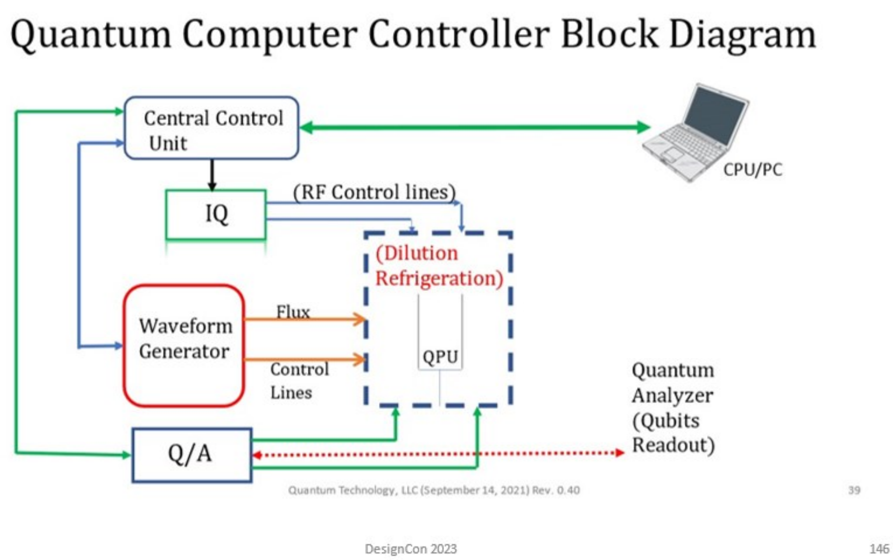


Fig 2: Quantum Computer Controller Block diagram

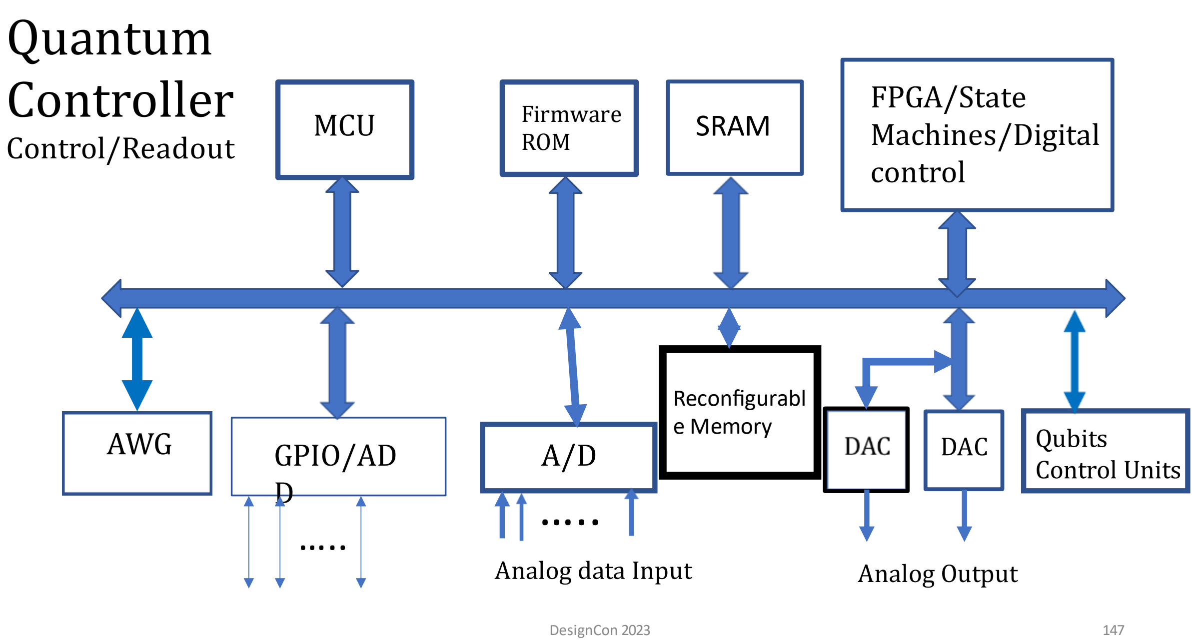


Fig 3. Quantum controller – Control/Readout

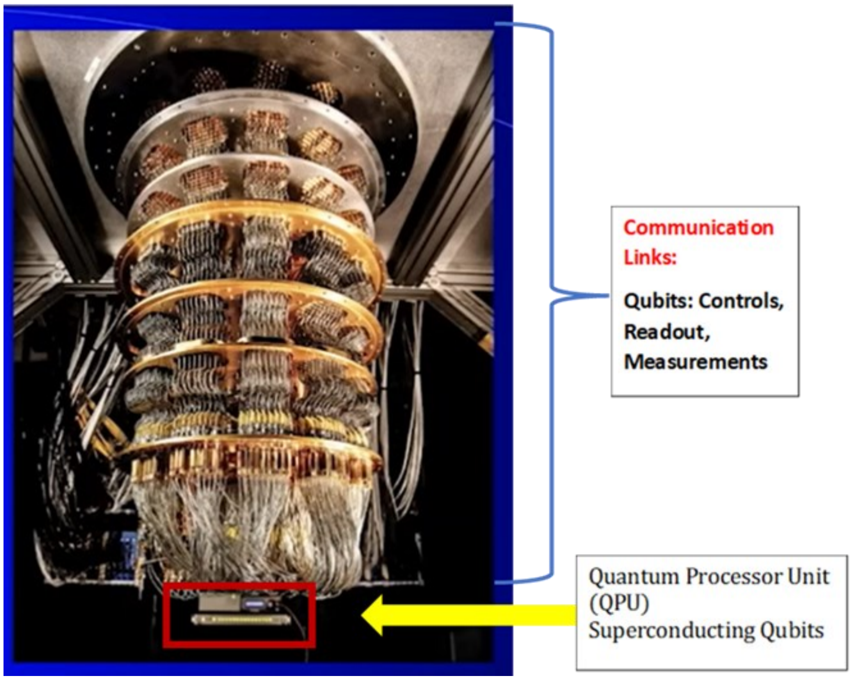


Fig 4: Typical Quantum Computer Communication Links,

Source: https://www.youtube.com/watch?v=_OjRClPzU6Y

Michio KaKu/The City University of New York

Talks at Google

How to scale up Quantum Computer? Quantum Notes: 12/22/2023

The US congress is on the way to pass National Quantum Initiative Reauthorization Act.

The NQI provides significant research and development funding to the Quantum fields; the world is competing the first to develop a general-purpose Quantum Computer. The consensus is that a quantum computer may require 1 million qubits.

To achieve such large scales of Quantum Computers requires completely new and stable Qubits and solid or advanced communication to connect the external (room temperature) control and read-out equipment to low-temperature Qubits (Superconductor Qubits). The electronic circuits have to be very accurate; many circuits have to be placed to close the low temperatures' qubits; the Qubits require many analog and digital circuits to control Qubits

• Large numbers of interconnect/entangled cables of electronic circuits to control and measure Qubits create a bottleneck for Quantum Computer to scale to Large Quantum Computers.

• Qubits must operate at low temperature, T= 10mK.

• Cryo-CMOS

• Control and measurement circuits operating under low temperatures.

• Low power

Qubit Processor Units (QPU) of Quantum Computer to achieve precise operations and maintain long coherence time, the QPU has to operate under extremely low temperature, 10mK. Quantum Computer's control circuits have to access individual qubits on the QPU; the connections between QPU and Quantum Control Circuits need many cables.
The communication between qubits to the digital controller, three major functional blocks, QPU interface to RF Analog circuits, then connect to digital control logics generate control waveform and detect Readout signals. For large-scale Quantum computers, the number of Qubits could reach millions on QPU and million-plus cables. It is an untenable engineering problem.
There are outstanding arguments for putting the readout circuit/control logic inside the dilution refrigerator to reduce the number of cables. Quantum Computer hardware has three major function blocks, which are QPU+RF analog circuits (communication)+ Digital Quantum Controller. We placed all three blocks inside the dilution refrigerator to reduce the tangled cables. We need Cryo-CMOS to design the RF analog circuits and Digital Quantum Controller.

Cryo-CMOS for Quantum Computer — A long road for million qubit computers:

The development works of Cryo-CMOS for Quantum Computer applications face many challenges. The challenges are to reevaluate and characterize the classical CMOS transistor's electrical parameters so that engineers can use the results to design Cryo-CMOS circuits on QPU. We understood that many electrical characteristics of Bulk CMOS are not suitable to use for in Quantum circuit (analog) design. The transistor's physical design and fabrication processes may require changes or new materials. New EDA tools support Cryo-CMOS transistors' parameters and layout tasks, etc.

A short history of Semiconductor:

We may learn a few things from the history of conventional semiconductor development works. In 1948, Bell Labs (Bardeen, Shockley, and Brattain) invented the Bipolar Junction Transistor; in 1959, R. Noyce of Intel's first true monolithic IC chip. 1959, M. Atalla and D. Kahng invented MOSFET at Bell Labs. 1963, Chih-Tang Sah and Frank Wanlass at Fairchild Semi. Invented CMOS. Robert H. Dennard of IBM paper: MOSFET scaling, Dennard Scaling, 1974. Intel's Tri-gate FinFET transistors, 2011.
One more datum point, Intel's 4004 (1971) had 2300 transistors, AM27C1024C, 1M CMOS EPROM (1985, AMD), i860 RISC (1989, Intel) were the million transistors chips, and i7-940 (2008) had 730 million transistors. It took four decades (40 years) to reach the million transistors. It will take many decades to build million qubits Quantum computer, a Universal Quantum Computer.
Engineers have enough resources; engineers can overcome Quantum Computing's challenges.

How to scale up?

To achieve a million qubits computer, a simple question is how to scale up. We currently don't have reasonable solutions, as Professor Michio Kaku/The City University of New York, Theoretical Physics, points out the issues of current qubits' physical scales matters.

Photonic Qubits: "Chinese version of Photons Quantum Computer with 113 detected photons (Qubits), The collection of mirrors and beam splitters is quite complicated."

Superconductor Qubits: How to untangle the cable connections between Room temperature electronics and low-temperature QPU (Quantum processor Units).
We must resolve the fundamental engineering issues and then move forward to scale up.

Quantum Modular Architectures:

IBM's new Quantum Processor Unit, 133 Qubits Heron, and IBM Quantum System Two are the company's first modular quantum computers, with IBM Heron processors and supporting control electronics. In conventional semiconductor technology, the modular approach is typical of architecture.

To scale up is engineering issues and technical issues.

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